This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the ICC 2008 proceedings.

Joint Power Control and Link Scheduling in Wireless Networks for Throughput Optimization

Liqun Fu, Soung Chang Liew, Jianwei Huang
Department of Information Engineering
The Chinese University of Hong Kong
Shatin, New Territories, Hong Kong
Email: {lqfu6,soung,jwhuang}@ie.cuhk.edu.hk

***Abstract*—This paper concerns the problem of finding the minimum-length TDMA frame of a power-controlled wireless network subject to traffic demands and SINR (signal-to-interference-plus-noise ratio) constraints. We formulate the general joint link scheduling and power control problem as an integer linear programming (ILP) problem. The linear relaxation of the ILP problem has been claimed to be NP-hard in the literature. We present a computationally efficient heuristic algorithm, called the Increasing Demand Greedy Scheduling (IDGS) algorithm, to solve the general ILP problem. In addition, we propose using a column generation (CG) method as an augmentation to IDGS to further improve its performance. Simulation results show that integration of IDGS and CG can achieve superior performance in terms of both algorithm run time and solution optimality.**

***Index Terms*—scheduling, power control, SINR constraints.**

I. Introduction

Due to their unconfined nature, signals transmitted over wireless links can mutually interfere with each other. To avoid detrimental interference, the transmissions on wireless links in the proximity of each other need to be properly scheduled. In an effort to boost network capacity, much research attention in recent years has been paid to wireless link scheduling *with power control* in TDMA (Time-Division Multiple Access) wireless networks (e.g., [1]–[6]).

This paper considers the following joint scheduling and power control problem. Given a wireless network with fixed topology and the traffic demands of the corresponding wireless links, how to find the minimum-length TDMA frame, the corresponding link transmission schedule, and the transmission powers of the links to deliver all the traffic such that all the transmissions are successful under the physical interference model? Minimizing frame length has the effect of maximizing network throughput in the network. Under the physical interference model, the signal-to-interference-plus-noise ratio (SINR) requirements of all receivers of concurrent transmissions must be satisfied. Through power control, we can mitigate the interference so that more wireless links can be scheduled to transmit simultaneously, hence decreasing the TDMA frame length.

This work was partially supported by the Competitive Earmarked Research Grant (Project Number 414507) established under the University Grant Committee of the Hong Kong Special Administrative Region, China.

Our work is closely related to the previous work in [2]–[4]. In [2], the authors examined the complexity of a similar problem. They proved that the linear relaxation of the problem is at least as hard as the MAX-SIR-MATCHING problem which they claimed is a hard problem. The strict proof of the NP-hardness of the problem is still not known. In [4], Jian *et al*. formulated the joint link scheduling and power control problem with fairness considerations. They also proposed a serial linear programming rounding (SLPR) heuristic to solve the problem. In [3], the authors formulated the power controlled minimum frame length scheduling problem as an mixed integer linear programming (MILP) problem. Unfortunately, this formulation requires exponential run time and is therefore computationally intractable. The authors also suggested a heuristic algorithm; however, they did not present any analysis or simulation-based evaluation of the proposed heuristic algorithm.

In this paper, we formulate the joint link scheduling and power control problem as an integer linear programming (ILP) problem, and use the Perron-Frobenius eigenvalue condition ([7], [8]) as a bridge to tie link scheduling and power control together in an integrated manner. Such an integration can streamline and expedite the optimization algorithm. The linear relaxation was previously formulated as a linear programming (LP) problem in [2]; however, no solution has been provided to solve the general LP problem. The main difficulty in solving both the LP and the ILP problem is that the number of decision variables increases exponentially with the number of links in the network. In this paper, we are able to propose solutions to solve the general ILP problem. We first present a simple and computational efficient heuristic algorithm, called Increasing Demand Greedy Scheduling (IDGS) algorithm. We also propose an algorithm based on the column generation (CG) method to further improve the performance of IDGS algorithm. The column generation method is a well known technique for solving LP problem with a huge number of variables [9], [10]. For the problem considered here, we found that the selection of the initial basis for CG is crucial to both solution optimality and run time efficiency. Interestingly, the solution of IDGS constitutes a good initial basis for CG. Such an integrated CG-IDGS algorithm can achieve superiority in terms of both algorithm run time and optimality of the solution.

The rest of the paper is organized as follows. In section II, we describe the wireless network model. The minimum frame length scheduling problem with power control is formulated in section III. We introduce the IDGS algorithm in section IV, and propose the column generation method as an augmentation to the IDGS algorithm in section V. The simulation results are shown in section VI. Section VII concludes this paper.

II. The Network Model

We consider a network with several transmission links which constitute a matching, defined as follows:

Definition 1: A matching M in a network is a set of links such that no two links in M share the same node.

Let I denote the index set of the links in matching M. Let $\{T_i : i \in I\}$ and $\{R_i : i \in I\}$ denote the set of transmitting nodes and the set of receiving nodes of the links in the matching M, respectively. Given a matching $M = \{(T_1, R_1), (T_2, R_2), \cdots, (T_{|M|}, R_{|M|})\}$, let $G(T_i, R_j)$ denote the path gain from transmitting node T_i to receiving node R_j. The average noise power at the receiving node R_i is denoted as N_i.

Definition 2: A sub-matching of a matching M is M with zero or more links removed.

Let $S = \{(T_{e1}, R_{e1}), (T_{e2}, R_{e2}), \cdots, (T_{e|S|}, R_{e|S|})\}$ denote a sub-matching of matching M, where $e_i \in I$ with $1 \le i \le |S|$ denote the indices of the links in the sub-matching S. A sub-matching S is said to be feasible if the SINR requirements at each receiving node in S are satisfied.

Definition 3: A sub-matching $S = \{(T_{e1}, R_{e1}), (T_{e2}, R_{e2}), \cdots, (T_{e|S|}, R_{e|S|})\}$ is a feasible sub-matching if we can find a positive power vector $\boldsymbol{p} = (p_{e1}, p_2, \cdots, p_{e|S|})^T$ such that the SINR constraints at each receiving node in the set $\{R_{ei}, 1 \le i \le |S|\}$ are satisfied:

$$\frac{p_{ei} G(T_{ei}, R_{ei})}{N_{ei} + \sum_{j=1, j \neq i}^{|S|} p_{ej} G(T_{ej}, R_{ei})} \ge \gamma_0 \quad (1)$$

where the threshold value γ_0 is assumed common throughout the network.

By Perron-Frobenius Theorem [11], the problem of deciding whether a sub-matching is feasible or not becomes the Perron-Frobenius eigenvalue condition of a nonnegative matrix. The following theorem is a compilation of the theorems shown in [7] [12].

Theorem 1: Consider the sub-matching $S = \{(T_{e1}, R_{e1}), (T_{e2}, R_{e2}), \cdots, (T_{e|S|}, R_{e|S|})\}$. Define the relative-path-gain matrix as the following $|S| \times |S|$ nonnegative matrix

$$B = \begin{pmatrix} 0 & \frac{G(T_{e2}, R_{e1})}{G(T_{e1}, R_{e1})} & \cdots & \frac{G(T_{e|S|}, R_{e1})}{G(T_{e1}, R_{e1})} \\ \frac{G(T_{e1}, R_{e2})}{G(T_{e2}, R_{e2})} & 0 & \cdots & \frac{G(T_{e|S|}, R_{e2})}{G(T_{e2}, R_{e2})} \\ \vdots & \vdots & & \vdots \\ \frac{G(T_{e1}, R_{e|S|})}{G(T_{e|S|}, R_{e|S|})} & \frac{G(T_{e2}, R_{e|S|})}{G(T_{e|S|}, R_{e|S|})} & \cdots & 0 \end{pmatrix}. \quad (2)$$

Let $\rho(B)$ denote the largest real eigenvalue of B (which is also called Perron-Frobenius eigenvalue or spectral radius).

1) If noise can be neglected, the largest signal-to-interference ratio (SIR) which can be achieved simultaneously by all the receiving nodes in the sub-matching is given by

$$\gamma^* = \frac{1}{\rho(B)}. \quad (3)$$

The power vector $\boldsymbol{p}$ that achieves γ^* is the Perron eigenvector corresponding to $\rho(B)$.

2) If noise can not be ignored, then the γ^* of the noiseless case is the supremum of SINR achievable at all receiving nodes. Let $N = (N_{e1}, \cdots, N_{e|S|})$ denote the average noise power at the receivers. Given the required SINR threshold $\gamma_0 < \gamma^*$, the power vector $\boldsymbol{p}_N$ which achieves an SINR of at least γ_0 at all receivers in the nonzero noise case is given by

$$\boldsymbol{p}_N = c \cdot \boldsymbol{p}, \quad (4)$$

where $\boldsymbol{p} = (p_{e1}, \cdots, p_{e|S|})$ is any Perron eigenvector of matrix B, and multiplier c satisfies the following inequality:

$$c \ge \max_{i \in \{1, \ldots, |S|\}} \left\{ \frac{N_{ei}}{p_{ei} \cdot G(T_{ei}, R_{ei}) \cdot (\gamma_0^{-1} - \gamma^{*-1})} \right\} \quad (5)$$

We assume that the transmit power is unlimited and can be any positive real number. Theorem 1 not only provides an easy condition to check the feasibility of a sub-matching, but also provides a solution to the transmit powers of transmitting nodes. This can streamline and expedite the optimization algorithm if used properly. In particular, given a sub-matching, if the required SINR γ_0 is less than or equal to $\frac{1}{\rho(B)}$, the transmit power vector can be set according to (4) and (5) in the algorithm; otherwise, no matter how we tune the transmit powers, the links in the sub-matching can not be active simultaneously.

Definition 4: A feasible sub-matching is said to be maximal if it is not properly contained in any other feasible sub-matching.

III. Problem Formulation

Consider a matching $M = \{(T_1, R_1), (T_2, R_2), \cdots, (T_{|M|}, R_{|M|})\}$. We assume that there is only one single channel in the network. The channel is divided into frames, and each frame consists a number of time slots of the same fixed duration. Let f_i denote the traffic demand on link $(T_i, R_i), 1 \le i \le |M|$, which is the number of time slots to be assigned to it in each frame. Our objective is to find a minimum-length TDMA frame (the smallest number of time slots) such that the links which are scheduled to transmit simultaneously form a feasible sub-matching, and the traffic demand on each link is satisfied (i.e., link (T_i, R_i) is scheduled for at least f_i time slots in the frame).

We introduce the percentage cost penalty to describe the penalty of different algorithms compared to the optimal solution. Let L denote the frame length, where L is the total number of time slots in a frame. Let L_{opt} denote the optimal value of the frame length. The percentage cost penalty is defined as

$$P = \frac{L - L_{opt}}{L_{opt}} \times 100\%. \tag{6}$$

Let $E = \{E_j : 1 \le j \le |E|\}$ be the set of all the feasible sub-matchings of matching $\{(T_1, R_1), (T_2, R_2), \cdots, (T_{|M|}, R_{|M|})\}$. The number of time slots allocated to each feasible sub-matching $E_j, 1 \le j \le |E|$, is denoted as a non-negative integer variable u_j. We introduce an $|M| \times |E|$ incidence matrix Q with elements q_{ij} such that

$$q_{ij} = \begin{cases} 1, & \text{if link } i \text{ is in the feasible sub-matching } E_j, \\ 0, & \text{otherwise.} \end{cases}$$

Each column in Q indicates a feasible sub-matching which must satisfy the SINR constraints. Each column in Q corresponds to a relative-path-gain matrix defined in Theorem 1. Let $Q_j, 1 \le j \le |E|$, denote the jth column in matrix Q and B_{Q_j} denote the relative-path-gain matrix corresponding to the column Q_j, respectively. Based on Theorem 1, the SINR constraints become the Perron-Frobenius eigenvalue condition. For each column in Q, the following inequality must be satisfied:

$$\frac{1}{\rho\left(B_{Q_j}\right)} \ge \gamma_0, \quad 1 \le j \le |E|\,. \tag{7}$$

The minimum-frame-length scheduling problem with power control which satisfies the traffic demands and SINR constraints can be formulated as an integer linear programming (ILP) problem, as follows:

$$\begin{aligned} \min \quad & \boldsymbol{e}^T \boldsymbol{u} \\ s.t. \quad & Q\boldsymbol{u} \ge \boldsymbol{f} \\ & \frac{1}{\rho\left(B_{Q_j}\right)} \ge \gamma_0, \qquad 1 \le j \le |E| \\ & \boldsymbol{u} \ge 0 \\ & int. \quad \boldsymbol{u} \end{aligned} \tag{8}$$

where $\boldsymbol{e}$ represents a vector whose components are all equal to 1 and $\boldsymbol{u} = (u_1, u_2, \cdots, u_{|E|})^T$, $\boldsymbol{f} = (f_1, f_2, \cdots, f_{|M|})^T$.

The difficulty of the above problem is twofold. First, given a matching, there is no known polynomial-time algorithm for finding all the feasible sub-matchings or finding all the maximal feasible sub-matchings. Second, even if all the feasible sub-matchings could be found, their number $|E|$ can be huge and in general increases exponentially with the number of links in the network [2]. This means that even if we ignored the integer constraint so that the problem becomes a linear programming (LP) problem, the dimension of the LP could be exponentially large. This motivates us to propose the heuristic algorithm introduced in the next section.

IV. Heuristic Scheduling Algorithm

In this section, we present our computationally efficient heuristic algorithm, called the Increasing Demand Greedy Scheduling (IDGS) algorithm, to solve the ILP problem described in section III. The input of the IDGS algorithm is the relative-path-gain matrix B and the traffic demand vector $\boldsymbol{f}$ of the matching M. The output of the IDGS algorithm is the set of feasible sub-matchings E, the set of non-negative integers $\boldsymbol{u}$, and the set of transmission power vectors $\boldsymbol{p}_N$.

Algorithm 1 IDGS Algorithm

Require: matrix B, demand vector $\boldsymbol{f}$
1: Sort the links in increasing order based on the traffic demands $\{f_1, f_2, \cdots, f_{|M|}\}$.
2: Initialize $\mathcal{F}_1$:=all the links in the matching
3: **while** $\mathcal{F}_i \neq \emptyset$ **do**
4: $\quad j := |\mathcal{F}_i|$
5: $\quad$ **while** $j \ge 1$ **do**
6: $\quad\quad$ Set $E_i := \{l_{i1}\}$
7: $\quad\quad$ $E_i := E_i \cup \{l_{ij}\}$
8: $\quad\quad$ **if** $\frac{1}{\rho\left(B_{E_i}\right)} < \gamma_0$ **then**
9: $\quad\quad\quad$ $E_i := E_i \setminus \{l_{ij}\}$
10: $\quad\quad$ **end if**
11: $\quad\quad$ $j := j - 1$
12: $\quad$ **end while**
13: $\quad$ $u_i := f_{i1}$
14: $\quad$ $\boldsymbol{p}_{Ni} := c \cdot \boldsymbol{p}_i$
15: $\quad$ $\mathcal{F}_{i+1} := \mathcal{F}_i$
16: $\quad$ **for** each link $l_{ij} \in \mathcal{F}_i$ **do**
17: $\quad\quad$ **if** $l_{ij} \in E_i$ **then**
18: $\quad\quad\quad$ $f_{i+1,j} := f_{ij} - u_i$
19: $\quad\quad\quad$ **if** $f_{i+1,j} \le 0$ **then**
20: $\quad\quad\quad\quad$ $\mathcal{F}_{i+1} := \mathcal{F}_{i+1} \setminus \{l_{ij}\}$
21: $\quad\quad\quad$ **end if**
22: $\quad\quad$ **end if**
23: $\quad$ **end for**
24: $\quad$ $i := i + 1$
25: **end while**

In the first step, the links to be scheduled are sorted according to the traffic demands in increasing order. The task of the ith iteration of the outer while-loop (from line 3 to line 25) is to compute one feasible sub-matching (E_i), the number of time slots allocated to the feasible sub-matching (u_i), and the corresponding transmission power vector ($\boldsymbol{p}_{Ni}$). Let $\mathcal{F}_i$ be the set of remaining links to be scheduled in the ith iteration of the outer while-loop. Initially, the set $\mathcal{F}_1$ contains all links. Let f_i denote the demand vector of set $\mathcal{F}_i$.

The task of the inner while-loop (from line 5 to line 12) is to compose the ith feasible sub-matching E_i. The main idea of forming the feasible sub-matching E_i is that we want to allocate the minimum number of time slots to E_i, and E_i should contain the links with larger traffic demands. Let l_{ij} and f_{ij} denote the jth link in set $\mathcal{F}_i$ and the remaining traffic demand of link l_{ij}, respectively. We first put link l_{i1} that has the minimum traffic demand among the set $\mathcal{F}_i$ into E_i. Then we start from the last link which has the largest traffic demand within the set $\mathcal{F}_i$ and add one link to the set E_i at a time. The link added to the set E_i must satisfy the condition that the link and all the links already in the E_i form a feasible sub-matching. If adding a link will cause the Perron-Frobenius condition to be violated, we move on to the next link without adding it.

After the set E_i is formed, we allocate $u_i = f_{i1}$ time slots to set E_i. The transmission power vector of the links in set E_i equals $c \cdot \boldsymbol{p}_i$, where $\boldsymbol{p}_i$ is any Perron eigenvector of matrix B_{E_i} and c satisfies the inequality (5) shown in Theorem 1.

The task of the for-loop (from line 16 to line 13) is to form the set $\mathcal{F}_{i+1}$ (the links to be scheduled in the next iteration) and the updated traffic demand vector f_{i+1}. The traffic demand of each link l_{ij} in the set E_i is subtracted by u_i and the traffic demands of the other links remain the same. The set $\mathcal{F}_{i+1}$ is formed by removing the links whose updated traffic demands are less than or equal to 0 from the set $\mathcal{F}_i$.

IDGS continues until all the links are removed from the link list. In each iteration of the outer while-loop, at least the traffic demand of one link will be satisfied entirely. Thus the maximum number of iterations required is no longer than the number of links in the matching.

IDGS is computationally efficient and also gives a good solution to the general ILP problem. Presentation of the simulation results supporting this claim is deferred to section VI. Next, we present a column generation (CG) method, which when combined with IDGS, yields even better performance.

V. A Column Generation Method

As shown in section III, the number of feasible sub-matchings can be huge, so that forming the coefficient matrix Q in full is impractical. We now introduce a column generation (CG) solution which uses the (revised) simplex method to generate columns of Q as needed rather than in advance.

We remove the integer restrictions in the last row of (8) and use CG to solve the linear programming problem. CG decomposes the original LP problem into two different problems, a restricted master problem and a sub-problem. The restricted master problem is similar to the original LP problem except that only a subset $E' \subseteq E$ is considered. Let Q' denote the incidence matrix of E'. The restricted master problem is as follows:

$$\begin{aligned} min \quad & \boldsymbol{e}^T \boldsymbol{u} && (9)\\ s.t. \quad & Q' \boldsymbol{u} \geq \boldsymbol{f} \\ & \frac{1}{\rho\left(B_{Q'_j}\right)} \geq \gamma_0, \qquad 1 \leq j \leq |E'| \\ & \boldsymbol{u} \geq 0 \end{aligned}$$

An initial subset of feasible sub-matchings can be easily formed. We may let the ith sub-matching consist of link l_i only ($1 \leq i \leq |M|$) and none of the other links. Since there is only one link in each sub-matching and the transmission power is unlimited, the SNR constraints can always be satisfied. All these $|M|$ sub-matchings are feasible sub-matchings. The corresponding initial matrix is an identity matrix. Another possible initial subset is the feasible sub-matchings found by IDGS described in section IV. For our problem, the selection of the initial subset in the column generation algorithm is crucial in terms of both the algorithm performance and the algorithm run time. (Please refer to the simulation results in section VI.)

First we solve the restricted master problem (9). This gives a basis matrix D and an associated basic feasible solution $\bar{\boldsymbol{u}}$. Because the cost coefficient of every variable in the objective function is unity, the simplex multipliers become $\boldsymbol{\omega}^T = \boldsymbol{e}_D{}^T D^{-1}$, where every component of the vector $\boldsymbol{e}_D$ is equal to 1. As known from linear programming [9], the reduced costs are defined by:

$$\sigma_j = e_j - \boldsymbol{\omega}^T Q_j = 1 - \boldsymbol{\omega}^T Q_j \qquad (10)$$

where Q_j denotes the column which is in the matrix Q but not in matrix Q'. Instead of computing the reduced costs for all the columns which are not in the matrix Q' in the simplex method, we consider the problem of minimizing the reduced cost $\sigma_j = 1 - \boldsymbol{\omega}^T Q_j$. This is equivalent to maximizing $\boldsymbol{\omega}^T Q_j$. If the maximum is less than or equal to 1, all the reduced costs are nonnegative and the current basic feasible solution $\bar{\boldsymbol{u}}$ is the optimal solution of the original LP problem; otherwise, the reduced cost is negative and the corresponding column enters the matrix Q'.

The sub-problem of finding a feasible sub-matching that maximizes $\boldsymbol{\omega}^T Q_j$ can be formulated as a combinatorial optimization problem:

$$\begin{aligned} max \quad & \sum_{i=1}^{|M|} \omega_i y_i && (11)\\ s.t. \quad & \frac{1}{\rho\left(B_y\right)} \geq \gamma_0 \quad 1 \leq i \leq |M| && (12)\\ & y_i \in \{0,1\} \qquad 1 \leq i \leq |M| \end{aligned}$$

where ω_i is the ith component of the simplex multipliers and y_i is a binary variable that is 1 if link l_i is active, and 0 otherwise.

The sub-problem itself is a difficult one. The main difficulty lies in the first constraint in (12), the SINR constraint. We propose the following heuristic algorithm, called the Combined Sum Criterion Selection (CSCS) algorithm, to solve the sub-problem. The idea is to remove one link at one time until the required γ_0 is satisfied in the remaining active links. The CSCS algorithm is as follows:

1) First solve the problem of finding a sub-matching that maximizes $\sum_{i=1}^{|M|} \omega_i y_i$ without the SINR constraints (12). This step is very easy. We set $y_i = 1$ if $\omega_i \geq 0$ and set $y_i = 0$ if $\omega_i < 0$.
2) Compute the best $\gamma^*{}_y = \frac{1}{\rho(B_y)}$ corresponding to the sub-matching y obtained in step one. If $\gamma^*{}_y \geq \gamma_0$, the sub-matching y is a feasible sub-matching and the sub-problem is solved; otherwise proceed to step 3. The

TABLE I
SIMULATION RESULTS (THE NETWORK CONTAINS 15 LINKS)

Algorithms	Average number of time slots	Average percentage cost penalty	Number of incidences in which optimal solution is obtained	Number of incidences in which percentage cost penalty is within 10%	Average run time (second)
Exhaustive search	42.684	0%	1000	1000	4.2985
CG-identity	59.463	39.19%	96	198	0.076194
IDGS	48.328	13.69%	173	457	0.014935
CG-IDGS	45.828	7.60%	437	692	0.048009

transmission power vector of feasible sub-matching y is $c \cdot \boldsymbol{p}_y$, where $\boldsymbol{p}_y$ is the Perron eigenvector of the relative-path-gain matrix B_y and c satisfies the inequality (5) given in Theorem 1.

3) Remove one link from the active links. Set $y_i = 0$ for which the maximum of the row sums and the column sums of the relative-path-gain matrix B_y is maximized (combined sum criterion):

$$\max_i \left\{ \sum_j (B_y)_{ij}, \sum_k (B_y)_{ki} \right\} \tag{13}$$

Go to step 2.

The combined sum criterion comes from [8] which investigates the power control problem in cellular systems. The combined sum criterion seeks to maximize the lower bound of the γ^* in the next iteration.

The feasible sub-matchings found in the column generation method give a good subset of "interesting" feasible sub-matchings which can then be used to find an integer solution. Problem (8) becomes the following integer linear programming problem:

$$\begin{aligned} min \quad & \boldsymbol{e}^T \boldsymbol{u} \\ s.t. \quad & Q_c \boldsymbol{u} \geq \boldsymbol{f} \\ & \boldsymbol{u} \geq 0 \\ & int. \quad \boldsymbol{u} \end{aligned} \tag{14}$$

where Q_c contains only the set of feasible sub-matchings found in column generation. Since all the columns in Q_c are feasible sub-matchings, the SINR constraints can be removed. Therefore, we can use the well-known methods (e.g., branch and bound) for solving the linear integer programming problem to solve the above problem.

VI. SIMULATION RESULTS

In this section, we carry out simulations to evaluate the performances of the IDGS , CG initialized with the identity matrix (CG-identity), and CG initialized with the solution of IDGS (CG-IDGS). For comparison purposes, we also obtain the optimal solution by exhaustive search.

In our simulations, the locations of the transmitting nodes are uniformly distributed in a square area of $1000m \times 1000m$. The distance of each link ranges from 100 to 200 meters. The location of each receiving node is randomly chosen within a radius of 200 meters and outside a radius of 100 meters from the corresponding transmitting node. The large-scale path loss model with the typical path loss exponent of 4 is assumed. The required SINR threshold $\gamma_0 = 10dB$. The traffic demand of each link is a discrete random variable with 10 equally likely values $[1, 3, 5, \cdots, 19]$. The expected value is 10 time slots. The number of links in the network is no more than 17. When the number of links is beyond 17, the exponentially increasing run time of the exhaustive search for benchmarking becomes prohibitive. For each given number of links in the network, we present the results averaged over 1000 instances. We conduct our simulations on a computer with a 1.86 GHz CPU and 1 GB of RAM.

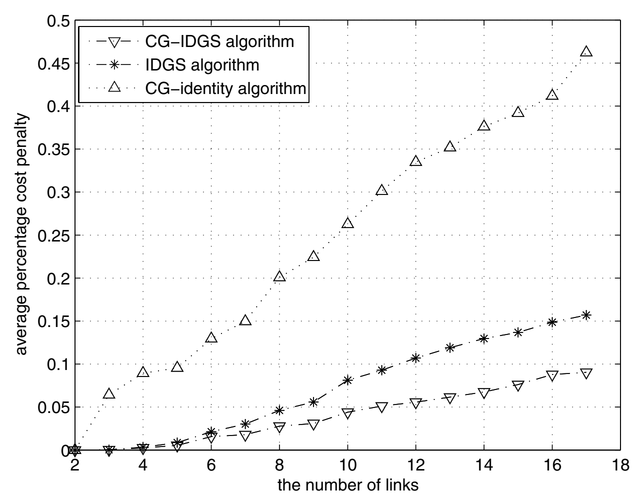

Fig. 1. Average Percentage Cost Penalty

The simulation results of the network containing 15 links are shown in Table I. Figure 1 shows the performance of average percentage cost penalty of the three algorithms as a function of the number of links in the network. We see that the performance of CG-identity is the worst among the three algorithms. For example, when there are 15 links, the average percentage cost penalty is 39.19% which is very high. Among the 1000 instances, only 96 reach the optimal solution. The average percentage cost penalty of CG-identity increases quickly with number of links (i.e., network density). In general, CG-identity can not ensure acceptable performance. The reason is that starting from the identity matrix as the initial solution, several very desirable feasible sub-matchings can not be found in the subsequent iterations of column generation. IDGS has better performance than CG-identity. The number of instances reaching the optimal solution increases to 173 and the average percentage cost penalty increases much more slowly with the number of links. CG-IDGS has the best performance among the three algorithms. The number of instances reaching the optimal solution is 437, more than twice and five times over those in IDGS and CG-identity, respectively. Also, the average percentage cost penalty increases more slowly with number of links in CG-IDGS.

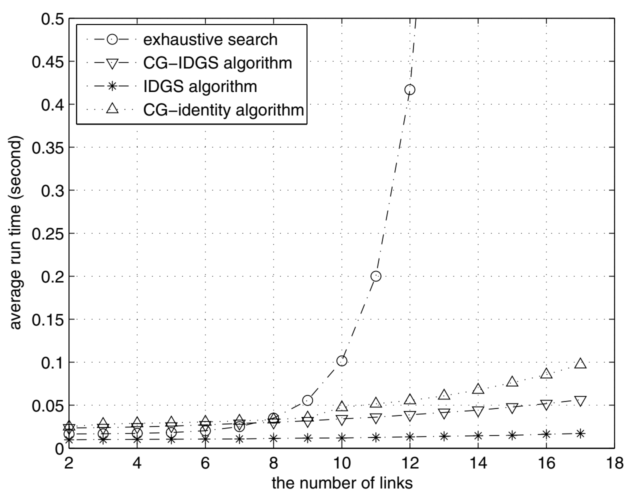

Fig. 2. Average Run Time

From the last column of Table I, we can see that the run time of all the three algorithms is very small compared with the run time of exhaustive search. Figure 2 plots run time versus number of links. Compared with the exponentially increasing run time of exhaustive search, the run times of the three algorithms increase much more slowly. The simplest algorithm, IDGS, consumes the shortest amount of time. CG-IDGS consumes a little more time. The reason for this small increase is that when starting with the feasible sub-matchings found in IDGS, CG terminates quickly after a few iterations. So much so that, CG-IDGS outperforms CG-identity in terms of run time. In short, CG-IDGS is an attractive algorithm in that superior solutions can be obtained with good run time performance.

VII. CONCLUSION

We have considered the minimum-frame-length scheduling problem in TDMA wireless networks with power control, subject to traffic demands and SINR constraints. We formulated the general joint link scheduling and power control problem as an ILP problem. The feasibility of a set of links under the SINR constraints can be checked by Perron-Frobenius eigenvalue condition. This turns out to be a rather useful condition for expediting the optimization algorithm. Accordingly, we have integrated the Perron-Frobenius eigenvalue condition into both the problem formulation and the proposed algorithms to improve their efficiency. We have presented a simple heuristic algorithm, Increasing Demand Greedy Scheduling (IDGS), to solve the general ILP problem. We have demonstrated that IDGS is computationally efficient and provides a good solution to the ILP problem. We further proposed an algorithm based on the column generation (CG) method. Although CG does not work well when it is initialized with an identity matrix initial solution (CG-identity), it works well and further improves the performance of IDGS if it is initialized with the IDGS solution (CG-IDGS). We showed that when the number of links is no more than 17 (in which the exponentially increasing run time of the exhaustive search for benchmarking can be afforded), the performance of the CG-IDGS is comparable to that of the optimal solution obtained by exhaustive search. In particular, simulation results show that the average cost penalty of the CG-IDGS relative to the optimal cost is below 10%, with an average run time less than 0.1 second.

In this paper, we have focused on wireless links that form a matching. We can easily extend the IDGS and the column generation algorithms for general wireless networks in which the links do not necessarily form a matching by redefining several elements of the relative-path-gain matrix B. Specifically, for the general setting, we set the element $B_{ij} = \infty$ if link $i \neq j$ and the transmitters and the receivers of link i and link j are not distinct, i.e., link i and link j have at least one node in common. The other elements of matrix B remain the same as defined in (2). After redefining the relative-path-gain matrix B, the links in the general network which satisfy the SINR constraints, i.e., the $\frac{1}{\rho(B)}$ is greater than or equal to the required SINR γ_0, will automatically form a matching. In fact, instead of setting $B_{ij} = \infty$, we could set B_{ij} equal to any value which is great than $\frac{1}{\gamma_0}$. Since the relative-path-gain matrix B is the input of the IDGS and column generation algorithms, there is no need to modify these two algorithms at all to schedule links in the general wireless network. We will discuss the detailed results of general networks in a future paper.

REFERENCES

[1] T. Elbatt and A. Ephremides, "Joint scheduling and power control for wireless ad hoc networks", *IEEE Trans. on Wireless Communications*, Vol.3, pp. 74-85, Jan. 2004.

[2] S. A. Borbash and A. Ephremides, "Wireless link scheduling with power control and SINR constraints", *IEEE Trans. Information Theory*, vol. 52, no. 11, pp. 5106-5111, Nov. 2006.

[3] A. K. Das, R. J. Marks, P. Arabshahi and A. Gray, “Power controlled minimum frame length scheduling in TDMA wireless networks with sectored antennas”, *Proc. IEEE Infocom*, pp. 1782-1793, 2005.

[4] J. Tang, G. L. Xue, C. Chandler and W. Zhang, “Link scheduling with power control for throughput enhancement in multihop wireless networks”, *IEEE Trans. Veh. Technol.*, vol. 55, no. 3, pp. 733-742, May 2006.

[5] G. Brar, D. M. Blough and P. Santi, “Computationally efficient scheduling with the physical interference model for throughput improvement in wireless mesh networks”, *Proc. ACM Mobicom*, pp. 2-13, 2006.

[6] R. L. Cruz and A. V. Santhanam, “Optimal routing, link scheduling and power control in multi-hop wireless networks”, *Proc. IEEE Infocom*, pp. 702-711, 2003.

[7] S. A. Grandhi, R. Vijayan, D. J. Goodman and J. Zander, “Centralized power control in cellular radio systems”, *IEEE Trans. Veh. Technol.*, vol. 42, no. 6, pp. 466-468, Nov. 1993.

[8] J. Zander, “Performance of optimum transmitter power control in cellular radio systems”, *IEEE Trans. Veh. Technol.*, vol. 41, no. 1, pp. 57-62, Nov. 1992.

[9] D. Bertsimas and J.N. Tsitsiklis, *Introduction to Linear Optimization*, Athena Scientific, 1997.

[10] M. E. Lubbecke and J. Desrosiers, “Selected Topics in Column Generation”, *Operations Research*, vol. 53, no. 6, Nov.-Dec. 2005.

[11] R. A. Horn and C. R. Johnson, *Matrix Analysis*, New York: Cambridge Univ. Press, 1991.

[12] S. A. Borbash and A. Ephremides, “The feasibility of matchings in a wireless network”, *IEEE Trans. Information Theory*, vol. 52, no. 6, pp. 2749-2755, Nov. 2006.