# Temporal Starvation in CSMA Wireless Networks

Cai Hong Kai[†*], Soung Chang Liew[*]
[†]China Electronics Technology Group Corporation No.38 Research Institute
[*]Department of Information Engineering, The Chinese University of Hong Kong, Hong Kong
email:{chkai6, soung}@ie.cuhk.edu.hk

***Abstract*—It is well known that links in CSMA wireless networks are prone to starvation. Prior works focused almost exclusively on *equilibrium starvation*. In this paper, we show that links in CSMA wireless networks are also susceptible to *temporal starvation*. Specifically, although some links have good equilibrium throughputs and do not suffer from equilibrium starvation, they can still have no throughput for extended periods from time to time. For real-time applications such as VoIP and video streaming, it is desirable to understand and characterize temporal starvation in CSMA wireless networks. To this end, we develop a "trap theory" to analyze the temporal throughput fluctuations. Based on the trap theory, we can develop analytical tools for computing the "degrees of starvation" for CSMA networks to aid network design. For example, given a CSMA wireless network, we can determine whether it suffers from starvation, and if so, which links will starve. Furthermore, the likelihood and durations of temporal starvation can also be computed. We believe that the ability to identify and characterize temporal starvation as established in this paper will serve as an important first step toward the design of effective remedies for it.***

***Index Terms*—Starvation, CSMA, IEEE802.11.**

## I. Introduction

Starvation in communication networks is an undesirable phenomenon in which some users receive zero or close-to-zero throughputs. Wireless carrier-sense-multiple-access (CSMA) networks, such as Wi-Fi, are prone to starvation [1]–[9].

In CSMA networks, different stations compete with each other using the CSMA medium-access control (MAC) protocol. When a station hears its neighbors transmit, it will refrain from transmitting in order to avoid packet collisions.

If each station can hear all other stations, the competition for airtime usage is fair. However, if each station hears only a subset of the other stations, and different stations hear different subsets of stations, then unfairness can arise. In this paper, we refer to such CSMA networks as "non-all-inclusive" networks. The unfairness in non-all-inclusive CSMA networks can be to the extent that some stations are totally starved while other stations enjoy good throughputs. As shown in prior works [1]–[5], starvation can happen in many CSMA network topologies, even in the absence of hidden terminals [10].

There are two types of starvation in CSMA networks:

- Equilibrium Starvation — A link could be starved because it receives near-zero throughput all the time.
- Temporal Starvation — A link could be starved in the *temporal* sense: it may have good long-term average throughput, but its throughput is near zero for excessively long periods from time to time.

The study of equilibrium throughputs in prior works [1]–[5] could only capture equilibrium starvation. The analysis of the temporal behavior of CSMA network is particularly challenging and there has been little prior research on the temporal throughput fluctuations for the non-all-inclusive CSMA networks. However, with the increasing real-time applications over CSMA wireless networks (e.g., VoIP and video streaming), it is desirable to understand and characterize temporal starvation in them. This paper is devoted to a detailed quantitative study of temporal starvation in CSMA networks. To our knowledge, this paper is the first attempt to characterize temporal starvation analytically.

The project was partially supported by Direct Grants 2050436 and 2050464 of the Chinese University of Hong Kong.

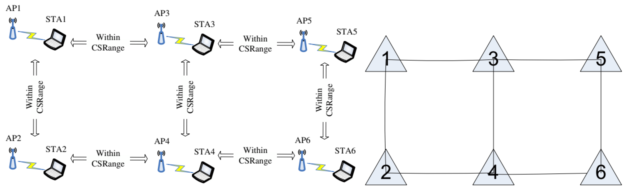


Fig. 1. An example network and its associated contention graph.

To characterize temporal starvation, we need to analyze the *transient* behavior of the underlying stochastic process of the CSMA protocol. We emphasize that by "temporal", we do not mean that the starvation is temporary or ephemeral in nature. Indeed, temporal starvation in CSMA wireless networks can be long-lasting.

Fig. 1 shows an example of temporal starvation. We have a small grid network consisting of six links. All the links have good long-term average throughputs; yet they suffer from temporal starvation, as described below.

The carrier-sensing relationships in the network are represented by the contention graph on the right of Fig. 1. In the contention graph, links are represented by vertices, and an edge joins two vertices if the transmitters of the two links can sense (hear) each other. Thus, in this network, when links 1, 4, and 5 transmit, links 2, 3, and 6 cannot transmit, and vice versa.

The normalized equilibrium throughput of each link in the network can be shown to be around 0.5 either by simulation or by analysis using the method in [5]. However, as shown by the simulation results presented in Fig. 2, the temporal throughputs of links vary drastically over time.

Fig. 2 plots the normalized throughputs versus time for links 1 and 2. Each data point is the throughput averaged over a window of one second. As can be seen, once a link gets access to the channel, it can transmit consecutively for a long time; on the other hand, once it loses the channel, it also has to wait for a long time before it enjoys good throughputs again.

The above example is a small network. Temporal starvation can be more severe for larger networks. For example, in an $N \times M$ grid network similar to that in Fig. 1, but with larger $N$ and $M$, the active and idle periods are much longer than those shown in Fig. 2.

In this paper, we propose a "trap theory" for the identification

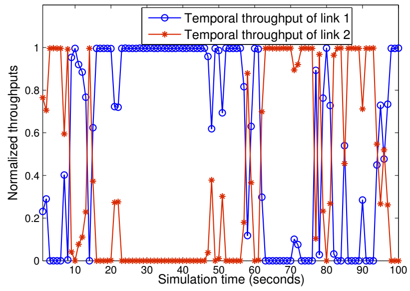


Fig. 2. Temporal throughputs measured over successive one-second intervals of links 1 and 2 in Fig. 1. Throughputs of other links exhibit similar fluctuations.

and characterization of temporal starvation. A trap is a subset of system states during which certain links receive zero or little throughputs; while the system evolves within the trap, these links suffer from temporal starvation. Based on the trap theory and the prior equilibrium analysis [5], we can construct computational tools to aid network design. For example, we can determine whether a given CSMA network suffers from starvation; if so, which links will starve, and whether the starvation is equilibrium or temporal in nature. Furthermore, for each link, the probability of temporal starvation and its duration can be characterized quantitatively. We believe the ability to identify and characterize starvation is an important first step toward finding the remedies to circumvent it.

***Related Work***

The equilibrium throughput of CSMA wireless networks has been well studied. Ref. [11] derived the equilibrium throughput of CSMA networks in which all links can sense all the other links. Refs. [2] and [5] investigated non-all-inclusive CSMA networks and showed that equilibrium throughputs of the links can be computed by modeling the network state as a time-reversible Markov chain. The temporal throughput fluctuations, however, were not considered.

Refs. [6]–[8] developed analytical models to evaluate the average transmission delay, delay jitter and the short-term unfairness in CSMA wireless networks. However, they only considered networks in which all links can hear each other.

Ref. [9] considered large CSMA wireless networks with 1D and 2D *regular* contention graphs. The border effects, fairness and phase transition phenomenon were investigated. Different from the regular networks studied in [9], this paper provides an analytical framework for characterizing temporal starvation in general CSMA wireless networks.

To save space, some derivations and results are omitted in this paper. They can be found in our technical report [12].

## II. System Model

We present an idealized version of the CSMA network (ICN) to capture the main features of the CSMA protocol responsible for the interaction and dependency among links. The ICN model was used in several prior investigations [2], [5], [9]. The correspondence between ICN and the IEEE 802.11 protocol [13] can be found in [5].

### A. The ICN model

In ICN, the carrier-sensing relationship among links is described by a contention graph in which each link is modeled as a vertex. Edges, on the other hand, model the carrier-sensing relationships among links. There is an edge between two vertices if the transmitters of the two associated links can sense each other.

At any time, a link is in one of two possible states, active or idle. A link is active if there is a data transmission between its two end nodes. Thanks to carrier sensing, any two links that can hear each other will refrain from being active at the same time. A link sees the channel as idle if and only if none of its neighbors is active.

In ICN, each link maintains a *backoff* timer, $C$, the initial value of which is a random variable with *arbitrary* distribution $f(t_{cd})$ and mean $E[t_{cd}]$. The timer value of the link decreases in a continuous manner with $dC/dt = -1$ as long as the link senses the channel as idle. If the channel is sensed busy, the countdown process is frozen and $dC/dt = 0$. When the channel becomes idle again, the countdown continues and $dC/dt = -1$ with $C$ initialized to the previous frozen value. When $C$ reaches 0, the link transmits a packet. The transmission duration is a random variable with *arbitrary* distribution $g(t_{tr})$ and mean $E[t_{tr}]$. After the transmission, the link resets $C$ to a new random value according to the distribution $f(t_{cd})$, and the process repeats. We define the *access intensity* of a link as the ratio of its mean transmission duration to its mean backoff time: $\rho = E[t_{tr}]/E[t_{cd}]$. In this paper, we will normalize time such that $E[t_{tr}] = 1$. That is, time is measured in units of mean packet duration. Thus, $\rho = 1/E[t_{cd}]$.

Let $x_i \in \{0, 1\}$ denote the state of link $i$, where $x_i = 1$ if link $i$ is active (transmitting) and $x_i = 0$ if link $i$ is idle (actively counting down or frozen). The overall **system state** of ICN is $s = x_1 x_2 ... x_N$, where $N$ is the number of links in the network. Note that $x_i$ and $x_j$ cannot both be 1 at the same time if links $i$ and $j$ are neighbors because (i) they can sense each other; and (ii) the probability of them counting down to zero and transmitting together is 0 under ICN (because the backoff time is a continuous random variable).

The collection of feasible states corresponds to the collection of independent sets of the contention graph. An independent set (IS) of a graph is a subset of vertices such that no edge joins any two of them [14]. For a particular feasible state $x_1 x_2 ... x_N$, link $i$ is in the corresponding IS if and only if $x_i = 1$. Thus, we may also denote the system state by the subset of active links in the state, e.g., $s = \{1, 4, 5\}$ represents a state in which links 1, 4 and 5 are active and the other links are idle. A maximal independent set (MaIS) is an IS that is not a subset of any other independent set [14], and a maximum independent set (MIS) is a largest maximal independent set [14]. Under an MaIS or an MIS, all non-active links are frozen, and none of them can become active.

As an example, Fig. 3 shows the state-transition diagram of the network in Fig. 1 under the ICN model. To avoid clutters, we have merged the two directional transitions between two states into one line. Each transition from left to right corresponds to the beginning of a transmission on one particular link, while the reverse transition corresponds to the ending of a transmission on the same link.

### B. Equilibrium Analysis

This part is a quick review of the result in [5], and the reader is referred to [5] for details. If we further assume that the transmission time and backoff time are exponentially distributed, then $s(t)$ is a time-reversible Markov process. For any pair of neighbor states in the continuous-time Markov

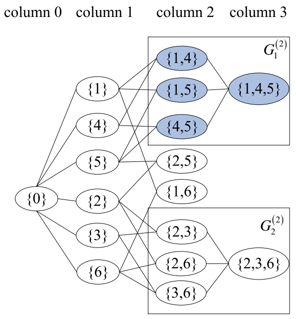


Fig. 3. The state-transition diagram of the network shown in Fig. 1. $G_1^{(2)}$ and $G_2^{(2)}$ are two traps.

chain, the transition from the left state to the right state occurs at rate $1/E[t_{cd}] = \rho$, and the transition from the right state to the left state occurs at rate $1/E[t_{tr}] = 1$. The stationary distribution of state $s$ can be computed by

$$P_s = \rho^n/Z, \forall s \in S^{(n)}, \quad \text{where} \quad Z = \sum\nolimits_n |S^{(n)}|\rho^n \tag{1}$$

In (1), $S^{(n)}$ is the subset of feasible states with $n$ active links and $Z$ is the normalization factor. The fraction of time during which link $i$ transmits is $Th_i = \sum_{s:x_i=1} P_s$. We will refer to $Th_i$ as the normalized throughput of link $i$.

Ref. [5] showed that (1) is in fact quite general and does not require the system state $s(t)$ to be Markovian. In particular, (1) is insensitive to the distributions of the transmission time and the backoff time, given the ratio of their mean $\rho$. In other words, (1) still holds even if the transmission time and the backoff time are not exponentially distributed.

## III. Traps and Temporal Starvation

This section gives the mathematical definition of traps and relates traps to temporal starvation in CSMA networks.

### A. *Definition of Traps*

Recall that for a non-MaIS state, the transition rate to a right neighbor state is $\rho$, and the transition rate to a left neighbor is 1. Note that the backoff countdown period is typically much smaller than the packet transmission duration in CSMA wireless networks (i.e., $\rho$ is large) and $\rho$ can be larger when TXOP [15] is increased to reduce the backoff countdown overhead. Large $\rho$ tends to push the system to states with more transmitting links. That is, in the state-transition diagram the movement from the right to the left is much more difficult than the movement from the left to the right. This could be seen from the relationship given in (1) as well, in which states with more transmitting links have higher probabilities through the factor $\rho^n$.

Before defining traps precisely, for illustration and motivation, let us look at the example of Fig.1. With respect to its state-transition diagram in Fig. 3, MIS $\{1,4,5\}$ and $\{2,3,6\}$ have the highest probabilities. Starting from either MIS, the process will next visit a state with one fewer transmitting link when it makes a transition. After that, the state may evolve back to the MIS (with rate $\rho$) or to a state with yet one additional idle link (with rate 1). However, large $\rho$ makes the movement to the left states a lot less likely. The system process tends to circulate among the subset of states composed of an MIS and its neighboring states. In particular, with large $\rho$, the system evolution will be anchored around the MIS, with departures from it soon drifting back to it. This will continue for a duration of time, depending on the "depth" of the trap (to be defined soon) and the value of $\rho$, until the system evolves to the other set of states anchored by the other MIS.

To isolate the two sets of states anchored around the two MIS, we could truncate the left two columns of the state-transition diagram in Fig. 3. We could then define the sets of states connected to MIS $\{1,4,5\}$ and MIS $\{2,3,6\}$ as two traps, respectively (i.e., the states enclosed in the two boxes in Fig. 3). Links $2,3,6$ suffer from starvation in the first trap, and links $1,4,5$ suffer from starvation in the second trap. We could use a *transient* analysis to analyze the time it takes for the system to evolve out of a trap, which sheds light on the duration of temporal starvation.

From Fig. 3, we see that a trap is a subset of "connected" states in which multiple links (e.g., links 1, 4 and 5) transmit and hog the channel for excessive time. While the system evolves within this subset of states, their neighboring links (e.g., links 2, 3 and 6) may get starved. Within the trap, the throughputs of these starved links may be much lower than their equilibrium throughputs. In this case, we say that temporal starvation occurs.

Moving beyond the above illustrating example, we now present the exact definition of traps in a general CSMA network. Let us denote the complete state-transition diagram of a CSMA network by $G$. In $G$, we arrange the states (vertices) such that the states with the same number of active links are in the same column. Label the column from left to right as $0,1,2,\cdots$ (i.e., the states in column $l$ have $l$ active links).

**Definition of the $l$-column truncated state-transition diagram:** The state-transition diagram with columns $0,1,2,\cdots,l-1$ truncated, denoted by $G^{(l)}$, will be referred to as the $l$-column truncated state-transition diagram. Each state in the leftmost column of $G_l$ has $l$ transmitting links [1].

**Definition of disconnected subgraphs and traps in $G^{(l)}$:** $G^{(l)}$ may consist of a number of subgraphs such that within each subgraph, all states are connected, but the states between the subgraphs are disconnected. Let $N_l$ denote the number of disconnected subgraphs in $G^{(l)}$, and $G_1^{(l)}, G_2^{(l)}, \cdots, G_{N_l}^{(l)}$ denote the subgraphs themselves. A subgraph $G_j^{(l)}, j \in \{1,...,N_l\}$, is said to be a *trap* if there are at least two columns in it [2].

**Procedure to identify traps**

As mentioned above, temporal starvation occurs within traps. To determine whether a given network suffers from temporal starvation, we need to study traps in its state-transition diagram. We now describe a procedure to decompose the system states into traps in a hierarchical manner (in general, there could be traps within a trap). In practice, this procedure could be automated by a computer program as part of a toolset to identify and analyze temporal starvation for a given CSMA network.

We use the network on the left of Fig. 4 as an illustrating

[1]Note that when we truncate a state (vertex), we also eliminate the transitions (edges) out of it and into it. If two states are retained in a truncated graph, the transitions between them remain intact.

[2]The reason for requiring $G_j^{(l)}$ to have at least two columns to qualify as a trap is as follows. A general property of ICN is that in $G$, there is no direct transition (edge) between two states of the same column. Thus, if $G_j^{(l)}$ has only one column, then it must have only one single state; otherwise, the condition that all states in $G_j^{(l)}$ are connected as defined above would not be fulfilled. This means that when the process enters $G_j^{(l)}$, with probability 1 the next state that the process will visit will be a state in the left of $G_j^{(l)}$. That is, regardless of $\rho$, the process will not get "trapped" in $G_j^{(l)}$ for long.

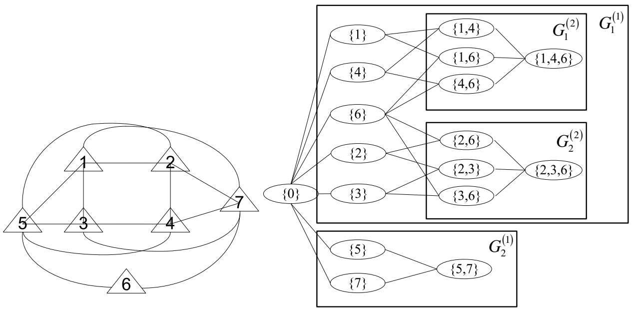


Fig. 4. An example network for illustrating the procedure to identify traps.

example. The state-transition diagram of the network is shown on the right of Fig. 4.

1) **Step 1:** We find the minimum $l$ such that $G^{(l)}$ consists of at least two disconnected subgraphs. Among the subgraphs $G_1^{(l)}, G_2^{(l)}, \cdots, G_{N_l}^{(l)}$, if none of them has at least two columns, then there is no trap in the network. Otherwise, we call those $G_j^{(l)}, j \in \{1, ..., N_l\}$, with at least two columns the first-level traps. For the example of Fig. 4, we have $l = 1$, and $N_l = 2$. Both $G_1^{(1)}$ and $G_2^{(1)}$ are first-level traps as shown on the right of Fig. 4.

**Depth of a trap:** For each trap, we define the depth of a trap as the maximum cardinality of the states in the trap minus $l$. Mathematically, the depth of a trap is

$$D\left(G_j^{(l)}\right) = \max_{s \in G_j^{(l)}} |s| - l \tag{2}$$

In other words, $D\left(G_j^{(l)}\right) + 1$ is the number of columns in $G_j^{(l)}$; and for $G_j^{(l)}$ to qualify as a trap, $D\left(G_j^{(l)}\right) \geq 1$.

For simplicity, when we refer to a trap in general, instead of writing $G_j^{(l)}$, we write $Tr$ with the understanding that there is an implicit $l$ and $j$ in $Tr$. Let $Th(i\,|\,Tr)$ denote the normalized throughput of link $i$ given that the process is within the trap. Mathematically, $Th(i\,|\,Tr)$ is a conditional probability:

$$\begin{aligned} &Th(i\,|\,Tr) = \Pr\{link\ i\ is\ active\ |the\ process\ is\ within\ Tr\} \\ &= \Pr\{s : x_i = 1|\, s \in Tr\} = \textstyle\sum_{s:x_i=1, s\in Tr} P_s \Big/ \sum_{s\in Tr} P_s \end{aligned} \tag{3}$$

where $P_s$ is given by (1).

We define the links which cannot receive a minimum target throughput while within the trap as the starving links of the trap, denoted by $S\,(Tr)$:

$$S\,(Tr) = \left\{ i | Th\,(i\,|Tr) < \overline{Th}_{\text{temp}} \right\} \tag{4}$$

where $\overline{Th}_{\text{temp}}$ is determined by the requirement of the applications running on top of the wireless network.

For our example in Fig. 4, we have two first-level traps: $G_1^{(1)}$ and $G_2^{(1)}$. $D\left(G_1^{(1)}\right) = 2$ and $D\left(G_2^{(1)}\right) = 1$. For any $\overline{Th}_{\text{temp}} > 0$, we have $S\left(G_1^{(1)}\right) =$ links $\{5, 7\}$ and $S\left(G_2^{(1)}\right) =$ links $\{1, 2, 3, 4, 6\}$.

2) **Step 2:** For each first-level trap, we increase $l$ further and check whether it can be further decomposed into a number of second-level traps.

For our example in Fig. 4, $G_2^{(1)}$ cannot be decomposed any further, while $G_1^{(1)}$ can be decomposed to two second-level traps: $G_1^{(2)}$ and $G_2^{(2)}$. $D\left(G_1^{(2)}\right) = 1$ and $D\left(G_2^{(2)}\right) = 1$; for any $\overline{Th}_{\text{temp}} > 0$, we have $S\left(G_1^{(2)}\right) =$ links$\{2, 3, 5, 7\}$ and $S\left(G_2^{(2)}\right) =$ links$\{1, 4, 5, 7\}$.

3) **Further Steps:** Similarly, we construct the third-level traps by decomposing the second-level traps. Repeat this procedure until all the newly formed traps cannot be decomposed further.

### *B. Definition of Temporal Starvation*

Next we relate the trap concept to temporal starvation. In this paper we define temporal starvation as follows:

**Definition of temporal starvation:** A link $i$ is said to suffer from temporal starvation if there is at least a trap $Tr$ with average trap duration larger than $T_{\text{target}}$ in which link $i$ gets throughputs below $\overline{Th}_{\text{temp}}$ within the trap (i.e., link $i \in S(Tr)$).

As will be argued in Section IV, the average duration of a trap is a function of both $D(Tr)$ and $\rho$. In particular, it grows exponentially with $D(Tr)$. That is, $D(Tr)$ is an important parameter characterizing the severity of the temporal starvation suffered by links in $S\,(Tr)$. Our procedure to identify temporal starvation presented in Section V is motivated by this result.

## IV. Analysis of Trap Duration

We characterize the mean trap duration by its ergodic sojourn time and study its properties. In particular, we obtain asymptotic analytical results for the trap duration for large $\rho$. When $\rho$ is not large, we propose a simple computation method by constructing an approximate Birth-Death process, from which we can obtain closed-form expression of ergodic sojourn time of traps. Interested readers are referred to Section V-D of [12] for more details.

### *A. Definition of Ergodic Sojourn Time of a Trap*

We now define the ergodic sojourn time of a trap, which provides a measure of the mean trap duration.

Write $B = G\backslash Tr$. All visits to a trap $Tr$ begin at some state within it. Assuming the system process is ergodic, we would like to derive the probability of a visit to $Tr$ beginning at state $s \in Tr$. Let $h_{Bs}$ be the average number of visits to $Tr$ per unit time that begins at state $s \in Tr$, defined as follows:

$$\begin{aligned} h_{Bs} &= \lim_{t\to\infty} [\text{the number of transitions from } B \text{ to } s \text{ in } (0,t)]\,/t \\ &= \textstyle\sum_{s'\in B} P_{s'} \upsilon_{s's} \end{aligned} \tag{5}$$

where $\upsilon_{s's}$ is the transition rate from state $s'$ to state $s$ in the complete continuous-time Markov chain $s\,(t)$.

When the system just arrives at the trap, the initial distribution is given by

$$P_s\,(0) = \begin{cases} h_{Bs} \Big/ \sum_{s'\in Tr} h_{Bs'} & s \in Tr \\ 0, \quad s \in B \end{cases} \tag{6}$$

**Definition of Ergodic sojourn time of a trap:** The ergodic sojourn time of a trap $Tr$ is defined as the time for the system process to evolve out of the trap given that the initial condition (6):

$$T_V\,(Tr) = \sum\nolimits_{s\in Tr} P_s\,(0)\, T_{sB} \tag{7}$$

where $T_{sB}$ is the expectation of the first passage time from a particular state $s$ within $Tr$ to the subset of the state space $B$.

In fact, a similar definition is used in [16] to characterize the expected sojourn time of visits to a group of states in general Markov chains.

It can be shown that the journey into the trap begins at any of the $|A_l|$ states in the leftmost column of the trap with equal probability (details can be found in [12]). Thus, (7) can be written as

$$T_V\,(Tr) = \sum\nolimits_{s\in Tr, |s|=l} T_{sB}/|A_l| \tag{8}$$

### B. Property of $T_{sB}$

We study the time it takes to exit the trap given that the system is in a particular state within the trap: $T_{sB}$.

We index the columns of a trap $Tr = G_i^{(l)}$ with respect to the overall state-transition diagram $G$. That is, column $l$ refers to the leftmost column, and column $l + d$ refers to the rightmost column, of $Tr$, where $d$ is its depth. Let $A_k$, $l \leq k \leq l + d$ denote the states in column $k$ of the trap. We have the following theorem:

*Theorem 1:* Consider a trap $Tr = G_i^{(l)}$ within the state-transition diagram of a CSMA wireless network. For any state $s \in Tr$ and $\rho > 1$, we have

$$T_{sB} = \beta\rho^d + o\left(\rho^d\right) \tag{9}$$

where $d$ is the depth of the trap, and $\beta = |A_{l+d}| \,/\, (l\,|A_l|)$; $|A_l|$ is the number of states in the leftmost column of $Tr$, and $|A_{l+d}|$ is the number of states in the rightmost column of $Tr$.

*Proof:* Theorem 1 is non-trivial to prove. We refer the reader to the Appendix A of [12] for the lengthy proof. ■

Theorem 1 indicates that starting with any state within the trap, the expected passage time to arrive at a state outside the trap is of order $\beta\rho^d$, where $\beta$ is a constant determined by the network topology and $d$ is the depth of the trap. Given a fixed network topology, $T_{sB}$ increases polynomially with $\rho$. Given a fixed $\rho$, $T_{sB}$ increases exponentially with $d$. We can see that for a large $\rho$ and a finite network, traps of higher depth are much more significant than traps of lower depth in terms of trap duration.

An interesting and significant observation of Theorem 1 is that for large $\rho$, the dominant term $\beta\rho^d$ in $T_{sB}$ is independent of the state $s$. Different states yield different $T_{sB}$ only through the term $o\left(\rho^d\right)$. This means that for large $\rho$, the duration of the trap depends only weakly on where the journey into the trap begins.

### C. Asymptotic results of $T_V(Tr)$

Combining Theorem 1 and (8), we have

$$T_V(Tr) = \beta\rho^d + o\left(\rho^d\right) \tag{10}$$

For large $\rho$, $T_V(Tr)$ is dominated by the term $\beta\rho^d$. For moderate $\rho$, we provide a simple method to approximate $T_V(Tr)$ in Section V-D of [12].

According to our definition of traps, the ergodic sojourn time of a trap provides a lower bound for the duration of temporal starvation. Once the system process evolves into a trap, on average the starving links of the trap will receive below-$\overline{Th}_{\text{temp}}$ throughputs for at least the duration of the trap. Furthermore, a starving link of a trap may starve even longer if the system returns to the trap without passing through states in which the link receives good throughputs.

## V. Analyzing Temporal Starvation Using Trap Theory

In this section, we propose the procedure to identify temporal starvation from traps and list the corresponding starving links. Besides the mean trap duration studied in Section IV, the severity of temporal starvation is further characterized by the probability of traps. One potential outcome of our analysis above is to construct a computational toolset to quantitatively characterize temporal starvation in a general CSMA wireless network.

### A. Procedure to Identify Temporal Starvation

Links suffer from temporal starvation within traps. The normalized link throughput within the traps can be computed using (3). To identify temporal starvation according to its definition in Section III-B, we need to examine the mean trap duration. Next we convert the target upperbound of average trap duration $T_{\text{target}}$ to reference trap depth $d_{\text{target}}$.

**Converting $T_{\text{target}}$ to $d_{\text{target}}$:** As can be seen in (10), the ergodic sojourn time of a trap is a function of both $\rho$ and $d$. Given $T_{\text{target}}$, we can determine the reference trap depth $d_{\text{target}}$ with respect to $\rho$ from (10) , i.e., $\beta\rho^{d_{\text{target}}} \approx T_{\text{target}} \to d_{\text{target}} \approx \log(T_{\text{target}}/\beta)/\log(\rho)$ (for $\rho$ that is not large, we could instead use the approximation provided in [12]). We then identify all the traps with depth no less than $d_{\text{target}}$ and list the links that suffer from temporal starvation.

Given the procedure to identify traps and $d_{\text{target}}$, the procedure to identify temporal starvation is quite straightforward: First, all the traps in the network are identified using the procedure described in Section III-A. We then go through all the traps with depth no less than $d_{\text{target}}$ and identify the links that suffer from temporal starvation.

Let us illustrate the procedure with the example in Fig. 4. For simplicity, we assume $d_{\text{target}} = 1$. For any $\overline{Th}_{\text{temp}} > 0$, links 1 and 4 suffer from temporal starvation in both traps $G_2^{(1)}$ and $G_2^{(2)}$; links 2 and 3, in both traps $G_2^{(1)}$ and $G_1^{(2)}$; links 5 and 7, in traps $G_1^{(1)}$, $G_1^{(2)}$ and $G_2^{(2)}$; link 6, in trap $G_2^{(1)}$.

For the above example, all links suffer from temporal starvation. However, their *probabilities* and *durations* of temporal starvation can be quite different. In general, the *significance* of a trap $Tr$ in terms of inducing temporal starvation on links $S(Tr)$ depends on two of its properties: the probability and the duration of $Tr$. The trap duration has been carefully analyzed in Section IV. We study the probability of a trap below.

### B. Probability of Traps

We define the probability of a trap as the stationary probability for the process to be within the trap:

$$\Pr\{Tr\} = \sum\nolimits_{s \in Tr} P_s \tag{11}$$

The probability of a trap $Tr$ characterizes how likely the links in $S(Tr)$ will suffer from temporal starvation because of $Tr$.

The probability of a trap can be directly obtained from (1). For our example in Fig. 4, we have

$$\begin{aligned}
&\Pr\{G_1^{(1)}\} = \left(5\rho + 6\rho^2 + 2\rho^3\right)/Z_0,\ \Pr\{G_2^{(1)}\} = \left(2\rho + \rho^2\right)/Z_0 \\
&\Pr\{G_1^{(2)}\} = \left(3\rho^2 + \rho^3\right)/Z_0,\ \Pr\{G_2^{(2)}\} = \left(3\rho^2 + \rho^3\right)/Z_0 \\
&Z_0 = 1 + 7\rho + 7\rho^2 + 2\rho^3
\end{aligned} \tag{12}$$

Given the value of $\rho$, we can compute the probability each link suffers from temporal starvation using (12).

### C. Temporal Analysis of General CSMA Networks

For general CSMA wireless networks, with theory and tools developed thus far, we can construct an analytical toolset to study the temporal behavior of throughputs. The tool can be implemented by a computer program for modest-size CSMA networks. The inputs to the program are the network topology in the form of a contention graph, the value of $\rho$ and $T_{\text{target}}$. The outputs of the program are as follows: 1) the list of starving links; 2) the list of traps in the network; 3) the probability of traps, and 4) the durations of traps.

Refer to our example in Fig. 4, the user inputs the contention graph shown on the left of Fig. 4, the value of $\rho$ and $T_{\text{target}}$. As on the right of the Fig. 4, the computer program produces the state-transition diagram together with identification of traps using the procedure described in Section III-A, upon which we obtain the lists of traps that incur temporal starvation with respect to $T_{\text{target}}$ in the network and the list of starving links. Then the user may want to find out the likelihood of links starvation and the durations of such starvation as follows.

All links in Fig. 4 suffer from temporal starvation assuming $d_{\text{target}} = 1$. Equation (11) characterizes the probabilities of traps in the network, from which we can compute the probabilities of the occurrence of temporal starvation for each link. Invoking (10), the ergodic sojourn time of traps can be computed as

$$\begin{aligned} &T_V\left(G_1^{(1)}\right) = \rho^2/5 + o\left(\rho^2\right), T_V\left(G_2^{(1)}\right) = \rho/2 + o\left(\rho\right) \\ &T_V\left(G_1^{(2)}\right) = T_V\left(G_2^{(2)}\right) = \rho/6 + o\left(\rho\right) \end{aligned} \tag{13}$$

When $\rho$ is large, the mean trap duration becomes rather large and hence the temporal starvation becomes more severe. In Fig. 4, links 1, 2, 3, 4 and 6 have good equilibrium throughputs; however, they still get starved when the system process evolves into trap $G_2^{(1)}$. To see this, we set up a simulation in which we initialized by letting links 5 and 7 transmit first (i.e., the system process starts within the trap $G_2^{(1)}$). Fig. 5 shows the temporal throughputs measured over successive 50-ms intervals. As can be seen, at the beginning period (0 ~ 250 ms), links 5 and 7 have the maximum throughputs while the other links receive zero throughputs, since the system process is within trap $G_2^{(1)}$. After that, the process evolves to trap $G_2^{(2)}$, in which links 1, 4, 5 and 7 get starved while links 2, 3 and 6 enjoy good throughputs. After 80*50 ms in the figure, the process evolves into trap $G_1^{(2)}$, in which links 2, 3, 5 and 7 starve. In the simulation, we observed that as time evolves, the system process transits among the three traps and all the links take turns to suffer from temporal starvation.

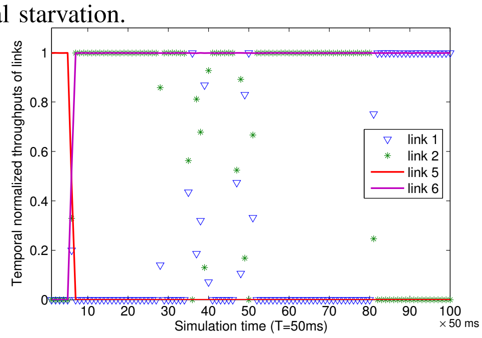


Fig. 5. Temporal throughputs of links 1, 2, 5 and 6 in the network of Fig. 4, and $\rho = 500$ measured over successive 50-ms intervals. Link 3, link 4 and link 7 have similar throughputs to that of link 2, link 1 and link 5, respectively.

In the network of Fig. 4, links 5 and 7 are the most prone to starvation and get starved in both traps $G_1^{(2)}$ and $G_2^{(2)}$. Most of the time links $\{1, 4\}$ and links $\{2, 3\}$ alternate to receive good and zero throughputs. Link 6, however, has good throughputs in both traps $G_1^{(2)}$ and $G_2^{(2)}$. Finally, links 1, 2, 3, 4 and 6 may get starved in trap $G_2^{(1)}$, although the probability of this starvation is small. The probability of link starvation and the mean trap duration can be computed by (12) and (13), respectively.

In general, our work allows the design of an automated computational tool to identify and quantitatively characterize starvation phenomenon in CSMA wireless networks. Given the state-transition diagram of the system, it is easy to determine computationally whether the truncated diagram $G^{(l)}$ is connected and then identify traps [14]. Hence, the complexity mainly lies in generating the state space of the system process as described in Section II-B. For modest-size CSMA wireless networks, we can quickly identify temporal starvation using our toolset described above. The complexity issue of large CSMA wireless networks will be tackled in our future studies.

## VI. Conclusion

This paper has proposed a framework called the trap theory for the study of temporal starvation in CSMA networks. The theory serves two functions: 1) it allows us to establish analytical results that provide insights on the dependencies of transient behavior of CSMA networks on the system parameters (e.g., how does access intensity $\rho$ affects temporal starvation); 2) it allows us to build computational tools to aid network design (e.g., a computer program can be written to determine whether a given CSMA network suffers from starvation, the degree of starvation, and the links that will be starved).

A goal of this paper is to enrich our understanding on the starvation phenomenon in CSMA wireless networks. Equilibrium throughput analysis cannot capture temporal starvation. The study of temporal starvation in this paper is a first step toward finding the solution for it.

## References


[1] P. Ng and S. Liew, "Throughput Analysis of IEEE802.11 Multi-hop Ad-hoc Networks," *IEEE/ACM Trans. Networking*, vol. 15, no. 2, pp. 309–322, 2007.

[2] X. Wang and K. Kar, "Throughput Modelling and Fairness Issues in CSMA/CA Based Ad-hoc Networks," in *Proceedings IEEE INFOCOM*, 2005.

[3] M. Garetto and et al., "Modeling Per-flow Throughput and Capturing Starvation in CSMA Multi-hop Wireless Networks," in *Proceedings IEEE INFOCOM*, 2006.

[4] K. Medepalli and F. A. Tobagi, "Towards Performance Modeling of IEEE 802.11 Based Wireless Networks: A Unified Framework and Its Applications," in *Proceedings IEEE INFOCOM*, 2006.

[5] S. Liew, C. Kai, J. Leung, and B. Wong, "Back-of-the-Envelope Computation of Throughput Distributions in CSMA Wireless Networks," *IEEE/ACM Trans. Networking*, vol. 9, no. 9, pp. 1319–1331, 2010.

[6] C. E. Koksal, H. Kassab, and H. Balakrishnan, "An analysis of short-term fairness in wireless media access protocols," in *Proceedings of the ACM SIGMETRICS*, 2000.

[7] Z. F. Li, S. Nandi, and A. K. Gupta, "Modeling the Short-term Unfairness of IEEE 802.11 in Presence of Hidden Terminals," *Elsevier, performance evaluation*, vol. 63, pp. 441–462, 2006.

[8] M. M. Carvalho and J. Garcia-Luna-Aceves, "Delay Analysis of IEEE 802.11 in Single-Hop Networks," in *Proceedings of the 11th IEEE International Conference on Network Protocols*, 2003.

[9] M. Durvy, O. Dousse, and P. Thiran, "Border Effects, Fairness, and Phase Transition in Large Wireless Networks," in *Proceedings IEEE INFOCOM*, 2008.

[10] L. Jiang and S. Liew, "Hidden-node Removal and Its Application in Cellular WiFi Networks," *IEEE/ACM Trans. Vehicular Technology*, vol. 56, no. 5, 2007.

[11] G. Bianchi, "Performance Analysis of the IEEE 802.11 Distributed Coordination Function," *IEEE Journal on Selected Areas in Communications*, vol. 18, no. 3, 2000.

[12] C. Kai and S. Liew, "Temporal Starvation in CSMA Wireless Networks," *Techinical report*, 2010, available at http://arxiv.org/ftp/arxiv/papers/1009/1009.3415.pdf.

[13] IEEE802.11-1997, IEEE 802.11 Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications.

[14] R. Diestel, *Graph Theory*. Springer-Verlag, Heidelberg, 2010.

[15] IEEE802.11e-2005, IEEE 802.11 Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications: Medium Access Control (MAC) Quality of Service Enhancements.

[16] J. Keilson, *Markov Chain Models-Rarity and Exponentiality*. Springer-Verlag, 1979.