Colouring Link-directional Interference Graphs in Wireless Ad Hoc Networks

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Abstract—**In this paper, we clarify inter-link interference in wireless ad-hoc networks by using link-directional interference graphs (l-graph). Most of the interference graphs in the literature simply model the DATA and ACK traffic of a link by a single vertex. They fail to capture the link-directionalities. In fact, some instances of directional traffic can actually transmit simultaneously but are prohibited by the interference graphs. Thus, in our link-directional interference graph, a link is represented by two vertices, one for DATA traffic and the other for ACK traffic. We then apply a colouring algorithm in the l-graphs. The colouring results provide insights in order to boost network capacities in TDMA or FDMA ad-hoc networks. We show that the network capacities can be improved by 100% in a triangular topology and 33% in a lattice topology. Simulations also show that a distributed dual channel protocol assigning channels according to link-directionalities can boost the capacities by 70% in large-scale random networks. We believe this is a first paper in the literature to take into account link-directionalities in interference graphs.**

Index Terms—**Wireless networks, Ad hoc networks, Interference graphs, Colouring algorithms, Capacity**

I. Introduction

In wireless ad-hoc networks, two wireless nodes (e.g. nodes X and Y) can communicate if they are located within the transmission range (TxRange) of each other. A wireless link (e.g. link X-Y) can then be established between these two nodes. However, when a third wireless node (e.g. node Z) placed inside the interference ranges of node X and/or node Y is transmitting, link X-Y can not transmit at the same time. Otherwise, collisions can occur and this degrades the network capacity. Link interference graphs can then be used to model these interference relationships between links. Link vertices are connected by interference edges if they cannot transmit simultaneously in the same channel. In order to eliminate these interference, colouring algorithms are applied to interference graphs for assigning orthogonal transmission channels or scheduling transmissions.

Many wireless network protocols in the literature are designed based on the interference graphs, for example, [1] [2] [3]. Reference [1] proposed a distributed algorithm to determine max-min fair share for links according to the interference graphs of ad-hoc networks. The algorithm is then integrated with the IEEE 802.11 protocol to achieve fairness between wireless links. Reference [2] introduced an approach for access points to allocate transmission channels in WLANs based on the colouring results of interference graphs. Simulations showed that the scheme can achieve higher network throughputs than random channel allocations.

The link interference graphs in the literature, however, fail to capture the link directionality. In data ad-hoc networks, a receiver node replies an ACK to a transmitter node after receiving a DATA packet. This generates reverse traffic in the link. A single vertex in a link interference graph for a link is insufficient to capture the interference relationships between links. In fact, some directional traffic can actually transmit simultaneously but are prohibited by the interference graphs (l-graph). Thus, in this paper, we propose a novel link-directional interference graph. By colouring link-directional interference graphs, we can allocate channels or schedule transmissions to improve network capacities.

The rest of this paper is organized as follows. In Section II, we review the physical interference graph model. Section III extends the model for networks with ARQ (Automatic Repeat-request). In Section IV, we present the link-directionality interference graph to clarify the interference relationships between directional links. A colouring algorithm is then applied to the link-directionality interference graph in order to schedule transmissions and allocate channels for TDMA and FDMA networks. Simulations in Section V show our schemes can significantly improve network capacities.

II. Physical Interference Graph (P-graph)

Reference [4] modeled the physical interference between links by two inequalities. Consider two links, links 1 and 2. Let T_i and R_i be the transmitter and receiver of link i, respectively. Link 2 can interfere with link 1 if

$$|X_{T2} - X_{R1}| < (1+\Delta)|X_{T1} - X_{R1}| \tag{1}$$

and link 1 can interfere with link 2 if

$$|X_{T1} - X_{R2}| < (1+\Delta)|X_{T2} - X_{R2}| \tag{2}$$

where X_i is the location of node i, and $\Delta > 0$ is related to a power margin. The physical interference range of a link i is then

$$PInRange_i = (1+\Delta)|X_{Ti} - X_{Ri}|$$

For example, in a two-ray propagation model and the capture threshold is set to be 10dB [5], $\Delta = 0.78$. Simultaneous transmissions on links 1 and 2 are not possible if either (1) or (2) is true. In other words, when T_1 (T_2) is located within the

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$PInRange_2$ ($PInRange_1$) of R_2 (R_1), links 1 and 2 cannot transmit concurrently. Otherwise, collisions will occur.

These two physical constraints can also be graphically represented by constructing physical interference graphs (p-graph) of wireless networks. In a p-graph, a vertex represents a wireless link. There is an edge between vertices 1 and 2 if (1) or (2) is satisfied. Figure 1 shows an example of mapping a network topology to a p-graph. Links P and Q can transmit simultaneously since T_P and T_Q are outside the $PInRanges$ of R_Q and R_P. On the other hand, link R cannot transmit at the same time with links P or Q since T_P and T_Q are within the $PInRange$ of R_R . Thus link R and links P and/or Q must take turns to transmit.

Based on the max-min fair share assignment proposed by [1], the expected capacity of the network can be obtained from the p-graph. The algorithm includes the following steps:

1. Separate the graph into cliques with initial capacities set as 1.
2. For the clique with the smallest value of capacity to number of vertices ratio, assign a fair share to all vertices in that clique.
3. These vertices are then removed from the graph and the capacities of remaining cliques are updated.

The steps 2-3 are repeated until all vertices are assigned with a fair share. Here we refer the interested reader to [1] for the details of the algorithm.

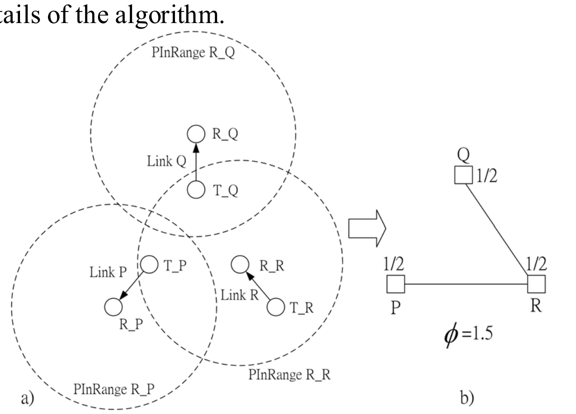

Figure 1. Mapping of a network topology a) to b) physical interference graph

Let ϕ be the sum of the "fair shares" of all vertices in the graph and C be the maximum throughput achieved by an isolated wireless link. The capacity of the network is then $C_{PHY} = \phi \cdot C$. All vertices in Fig. 1 are assigned by the algorithm with a fair share of 1/2. Thus, $C_{PHY} = 1.5C$ because links P and/or Q and link R each uses half of the total time for transmissions.

This capacity bound, C_{PHY} , however, is hardly achieved in wireless ad-hoc networks due to two factors: 1) symmetric traffic generated by replying ACKs and 2) protocol constraints.

For 1), in many wireless medium access protocols (e.g. IEEE 802.11), the receiver (R_i) always replies an acknowledgement packet (ACK) back to the transmitter (T_i) after successfully receiving a DATA packet. This can ensure the information integrity, particularly for data networks. Consider Fig. 1 again. When there are ACKs replying from the receivers (R_P and R_Q) to the transmitters (T_P and T_Q), links P and Q can no longer transmit simultaneously. This is because T_P (T_Q) are within the $PInRange$ of T_Q (T_P). When R_P (R_Q) is replying an ACK to T_P (T_Q) and T_Q (T_P) is sending DATA, a collision will occur at T_P (T_Q). Since T_P (T_Q) fails to receive the corresponding ACK of the transmitted DATA, the medium access protocol (e.g. IEEE 802.11) may treat it as a collision and retransmit the DATA packet. Therefore, links P and Q have to take turns to transmit and fail to reach the maximum achievable capacity, $C_{PHY} = 1.5C$.

For 2), wireless medium access protocols may limit the spatial reuse by imposing unnecessary protocol constraints. For example, reference [3] showed that simultaneous transmissions permitted by physical constraints (1) and (2) can be restricted by the virtual carrier sensing mechanism of the IEEE 802.11 protocol in ad-hoc networks. This is also well-known as the exposed-node problem. In addition, protocols may allow transmissions that will actually cause collisions (e.g. the hidden-node problem). These hinder protocols to achieve their expected performances.

In this paper, we will propose solutions for factor 1) so as to boost the capacities of networks. For factor 2), we refer the interested reader to references [3] and [6] in which solutions were proposed to modify the 802.11 MAC protocol for removing protocol constraints.

III. ARQ PHYSICAL INTERFERENCE GRAPH (ARQ P-GRAPH)

To take into account the symmetric traffic induced by replying ACKs, we need six additional inequalities for the physical constraints. When a link is transmitting ACK, its transmitter becomes a receiver and may collide with the DATA or ACK of other links. Thus, collisions due to the ARQ could occur if one of the following is true [3]:

$$\left|X_{R2} - X_{R1}\right| < (1+\Delta)\left|X_{T1} - X_{R1}\right| \qquad (3)$$

$$\left|X_{T1} - X_{T2}\right| < (1+\Delta)\left|X_{R2} - X_{T2}\right| \qquad (4)$$

$$\left|X_{T2} - X_{T1}\right| < (1+\Delta)\left|X_{R1} - X_{T1}\right| \qquad (5)$$

$$\left|X_{R1} - X_{R2}\right| < (1+\Delta)\left|X_{T2} - X_{R2}\right| \qquad (6)$$

$$\left|X_{R2} - X_{T1}\right| < (1+\Delta)\left|X_{R1} - X_{T1}\right| \qquad (7)$$

$$\left|X_{R1} - X_{T2}\right| < (1+\Delta)\left|X_{R2} - X_{T2}\right| \qquad (8)$$

As examples, figures 2a and 2b show the scenarios of inequalities (3) and (8) respectively. In Fig. 2a, if T_1 is sending DATA and R_2 is sending ACK at the same time, there will be a collision at R_1. Similarly, in Fig 2b, if R_1 and R_2 are sending ACKs at the same time, a collision will occur at T_2 . Thus, T_2 cannot receive the ACK successfully and will re-transmit DATA to R_2.

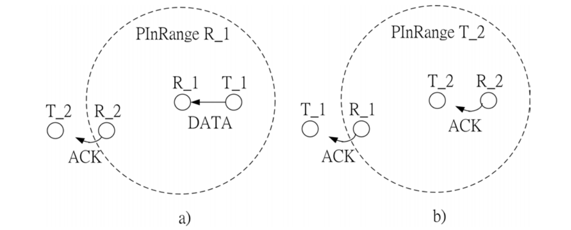

Figure 2. Network scenarios represented by inequalities a) (3) and b) (8)

We can then add inequalities (3)-(8) in constructing the ARQ p-graphs. Figure 3b shows the ARQ p-graph of the network topology in Fig. 1. Since T_P and T_Q are within the *PInRanges* of T_Q and T_P (inequalities (4) and (5) are true) respectively, there is now an edge between vertices P and Q which means links P and Q cannot transmit simultaneously. Therefore, links P, Q and R must take turns to transmit. In addition, the transmissions of ACKs take parts of the total transmission time. The capacity of a network with ARQ is then

$$C_{ARQ} = \frac{T_{DATA}}{T_{DATA} + T_{ACK}} \cdot \phi \cdot C$$

where T_{DATA} is the time for transmitting a DATA packet while T_{ACK} is the time for sending an ACK packet. In this paper, we assume the data rate is 11Mbps and the basic rate for ACK is 2Mbps. DATA size is fixed at 500 bytes and ACK is 30 bytes. With these settings,

$$C_{ARQ} = 0.75 \cdot \phi \cdot C$$

Here the protocol overheads are not taken into account since we have not specified our model to a particular protocol. As shown in Fig. 3, $C_{ARQ} = 0.75C$. Comparing C_{ARQ} with C_{PHY}, the capacity is drastically degraded from 1.5C to 0.75C.

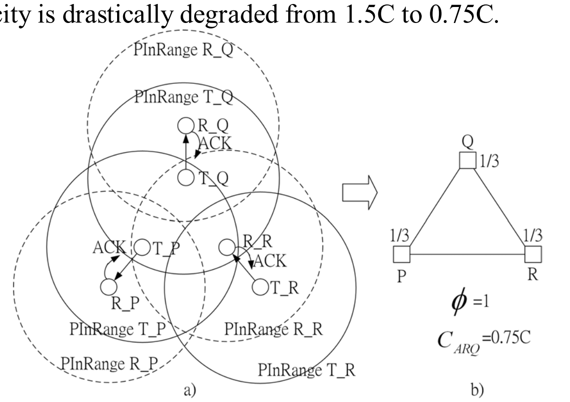

Figure 3. Mapping of a network topology with ARQ a) to b) ARQ physical interference graph

IV. Link-directional Interference graph (L-graph)

To clarify the interference relationships between links, we can further separate the vertices in ARQ p-graph into directional vertices based on their link-directionalities. In a link-directional interference graph (l-graph), an arrow-shape vertex represents a wireless link with the arrowhead pointing toward the receiver. Therefore, there are two vertices for a link with ARQ in a l-graph. One vertex represents the DATA traffic while the other represents the ACK traffic. We use the name of the destination node of a directional link as the name of the directional vertex. For example, the DATA traffic from T_Q to R_Q is represented by the directional vertex R_Q in the l-graph. Figure 4 shows the l-graph of the network topology in Fig. 1. There is an edge between two directional vertices if one of the inequalities (1)-(8) is true. For example, vertex R_Q is connected to vertex T_P, thus links R_Q and T_P cannot transmit simultaneously. In other words, collisions will occur if link Q is sending DATA while link P is replying ACK at the same time. On the other hand, links R_Q (T_Q) and R_P (T_P) can transmit DATA (ACK) concurrently since there is no edge between vertices R_Q (T_Q) and R_P (T_P) in the l-graph. We can then improve the network capacity by allowing non-interfering links to transmit simultaneously. The construction of l-graph can give us a clearer picture of the interference relationships which provides a new insight in increasing the capacity by proper scheduling or channel allocations.

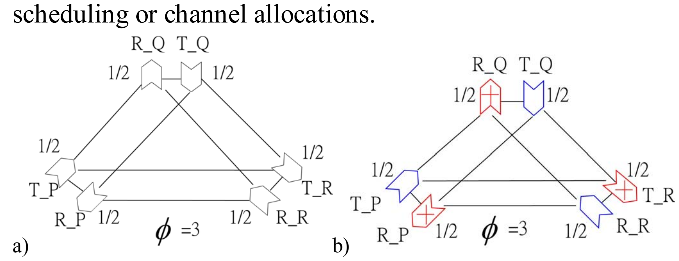

Figure 4. a) Link-directional interference graph of the network topology in Fig. 3b and b) its colouring result

Based on the insights obtained from l-grades, we can use 1) time division multiple access (TDMA) and 2) frequency division multiple access (FDMA) to increase network capacities. There are many TDMA and FDMA schemes proposed in the literature for ad-hoc networks. They can be modified with our proposed schemes. In this paper, we focus on the conceptual ideas of our schemes in scheduling transmissions and assigning channels instead of the protocol implementations. We refer the interested reader to [7][8][9][10] for the details of TDMA and FDMA protocols in ad-hoc networks.

For 1) TDMA, we have to figure out the groups of links that can transmit together. For 2) FDMA with two sub-channels, we have to assign one of the two sub-channels to vertices to eliminate interference between connected vertices. We apply a colouring algorithm in l-graphs with two colours and then use the colouring results for transmission scheduling or channel assignments. The colouring algorithm here is modified from the "degree of saturation" (DSATUR) algorithm proposed in [11] [2]. First, the "degree of saturation" is defined as the number of differently coloured neighbours of a vertex. The algorithm consists of the following steps:

1. Initialize the degrees of saturation of all vertices to 0.
2. Select the uncoloured vertex of highest degree of saturation. If more than one vertex have the same degree of saturation, choose the one with the highest number of uncoloured neighbours.
3. If the chosen vertex is connected to other vertices coloured with one of the two colours (say colour 1).

Colour the chosen vertex with another colour (say colour 2). Otherwise, randomly pick one of the two colours for the chosen vertex.

4. Update the degrees of saturation of the uncoloured vertices connected to the chosen vertex.

The steps 2-4 are repeated until all vertices are coloured.

Figure 4b shows the colouring of the l-graph. We will then schedule transmissions and assign channels for TDMA and FDMA according to the colouring results. If a vertex with colour 1 (2) is connected to another vertex in the same colour, these two vertices must take turns to transmit. Figure 5 shows an example. Since T_Z and R_Z are both within the *PInRanges* of R_X and T_X, links X and Z must take turns to transmit even though T_X and R_Z are in blue colour while R_X and T_Z are in red. Otherwise, collisions will occur.

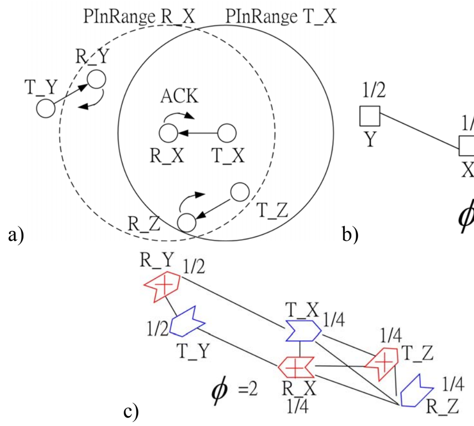

Figure 5. Mapping of a network topology a) to b) ARQ physical interference graph and c) coloured link-directional interference graph

For 1) TDMA, after colouring the l-graph, links assigned with the same colour can transmit concurrently without causing collisions. As shown in Fig. 6, links T_Q, T_P and R_R in Fig. 3a can transmit in one time slot while links R_Q, R_P and T_R can send packets in another time slot. In this way, all three links P, Q and R can transmit concurrently to boost the network capacity. Since the TDMA protocol needs two time slots for transmitting DATA and ACK of a complete transmission, the capacity of using TDMA is then

$$C_{Colour} = \frac{C}{2} \cdot \phi \tag{9}$$

Note that here a transmission of ACK still takes a whole time slot because ACK and DATA are now transmitting simultaneously. The link that sends ACK must wait for the completions of the transmissions of DATA by other vertices before sending DATA again. Otherwise, collisions may occur.

In the example of Fig. 3, the capacity is increased by up to $C_{Colour} = 1.5C$ which doubles the $C_{ARQ} = 0.75C$ of the network. However, TDMA requires synchronizations between links. This may not be easily achieved in distributed wireless ad-hoc networks unless a broadcast clock is available.

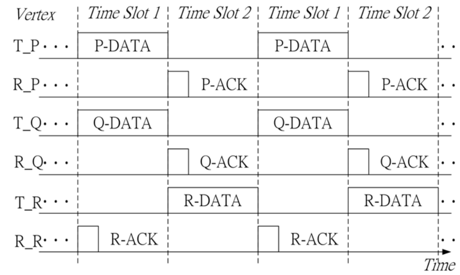

Figure 6. Scheduling the transmissions of links in the network topology of Fig. 3a according to the colouring result of l-graph in Fig. 4b

For 2) FDMA, we first divide the channel into two sub-channels and assign each of them for sending either DATA or ACK packets of a link. Consider the case of two sub-channels, sub-channels 1 and 2. DATA packets can be sent via sub-channel 1 while ACK packets can be transmitted in sub-channel 2. Note that here sub-channel assignments depend on link directionalities. Other links may use sub-channel 1 (or 2) for sending ACK while sub-channel 2 (or 1) for DATA.

This time, we use the colouring result to assign sub-channels instead of grouping simultaneous transmission links. After colouring the l-graph, links assigned with the same colour will use the same sub-channel for transmissions. In Fig. 4b, vertices T_P, T_Q and R_R use sub-channel 1 for transmissions while vertices R_P, R_Q and T_R use sub-channel 2 instead. In this way, T_P, T_Q and R_R (R_P, R_Q and T_R) can transmit simultaneously without causing collisions. Synchronizations between links are no longer needed since links use independent sub-channels for transmissions to avoid interference. Since the channel is divided into two sub-channels which in turn halves the transmission bit rate, the achievable capacity C_{Colour} is then the same as that achieved by TDMA in equation (9). This solution, however, does not require synchronizations between links which makes it more suitable for the distributive character of ad-hoc networks.

A. An example in a Lattice Topology

Figure 7a shows a 2x8 lattice topology. In ARQ p-graph as shown in Fig. 7b, neighbouring links (e.g. links A and B or links D and H) have to take turns to transmit since they are within the *PInRange* of each other. This yields $\phi = 4$ and $C_{ARQ} = 3C$. Figure 7c shows the coloured l-graph. With TDMA, blue coloured vertices can transmit at the same time in one time slot while vertices in red can use another time slot for concurrent transmissions. With FDMA, blue coloured vertices use sub-channel 1 while those in red use sub-channel 2 for transmissions. It does not require synchronizations between links which means red and blue vertices can transmit simultaneously as they use independent sub-channels. As a result, our proposed schemes yield $\phi = 8$ and $C_{Colour} = 4C$ which is 33% higher than C_{ARQ}.

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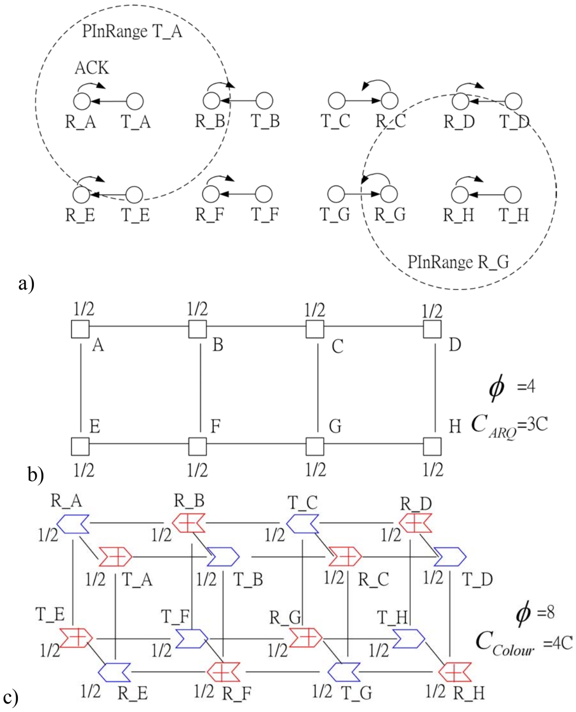

Figure 7. Mapping of a 2x8 lattice topology a) to b) ARQ physical interference graph and c) coloured link-directional interference graph

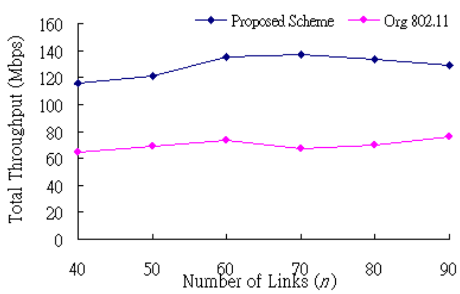

Figure 8. Total throughputs of random networks with n single-hop links

V. Simulation Results

In previous section, we have shown that our schemes can improve the capacities of the networks in Fig. 3a and Fig. 7 by 100% and 33% respectively. The TDMA and FDMA schemes we mentioned in Section IV, however, require the knowledge of network topology which may be difficult to obtain in distributed ad-hoc networks. Thus, in reference [12], we proposed a distributed link-directionality-based dual channel MAC protocol (DCP) for channel allocations. The protocol is modified from the original 802.11 MAC protocol and it attempts to seek opportunities for simultaneous transmissions based on the concept of link-directionality. Transmission channels of each link are assigned according to the availabilities of receptions of RTS and CTS from other links. Due to the space limitation, we refer the reader to [12] for details. We have implemented the DCP protocol [12] in the NS-2 [13] simulator. The data and basic rates are set at 12Mbps and 2Mbps respectively. The transmission range is 250m and the carrier-sensing range is 550m. In our simulations, all data sources are saturated UDP traffic stream with fixed packet size of 1460bytes.

Figure 8 shows the total throughputs of random topologies with n links. In each random network, n single-hop links are randomly placed inside a 3000m x 3000m square. In all cases, the capacity improvements achieve more than 70%. The throughput improvements, however, are highly dependent on the network topologies. In the best case when $n=70$, the DCP protocol boosts the total network throughput by 104%.

VI. Conclusion

This paper has presented the link-directional interference graph (l-graph) to refine the interference relationships between directional links. We have demonstrated that a colouring algorithm can be applied to l-graphs to provide insights in how to schedule transmissions or allocate channels in TDMA or FDMA ad-hoc networks respectively. We have showed that our schemes can boost the network capacities by 100% in a triangular topology and 33% in a lattice topology. Simulations also showed that more than 70% capacity improvements can be achieved with a distributed link-directionality-based dual channel protocol. We believe this is a first paper in the literature to take into account the link-directionalities in interference graphs.

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