MobiCom Poster Abstract: Capacity Improvement of Wireless Ad Hoc Networks with Directional Antennae

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I. INTRODUCTION

Network Capacity, which is limited by interference between simultaneous transmissions of neighboring links, is a fundamental performance metric for wireless ad hoc networks. Ref. [1] showed that the capacity of wireless ad hoc networks with omni-directional antennae does not scale well as the node density increases. Other related work ([2], [3], [4], etc.) demonstrated that the network capacity scales better with the use of directional antennae. Most previous investigations, however, were based on simplified but unrealizable models of antenna patterns. So far, there has not been a general framework for the analysis of network capacity when directional antennae of realizable generic patterns are used. Also, few papers have derived explicitly the impact of nulls and null-width in antenna pattern on network scalability. Specifically, the scale law of network capacity with general directional antennae has not yet been investigated. This paper addresses these issues. We study arbitrary and random networks (corresponding to the best case and random case respectively), where nodes with smart antennae are assumed to be located in a planar disk without mobility. We also point out how to generalize the result to that of generic directional antenna pattern. To conserve space here, we defer detailed analysis of the impact of null-width to the full paper.

II. GENERIC ANTENNA MODEL

A. *Pair-wise Physical Link Interference Model of Generic Directional Antenna*

Consider *n* nodes placed in a planar disk of area *A*. At any instant, there are at most $n_1 = \lfloor n/2 \rfloor$ transmitter-receiver pairs (links). We can represent the antenna power pattern, namely the normalized antenna gains of transmitter and receiver by G_T and G_R, as a function of azimuthal angle θ, where θ is the angle with respect to the direction at which the antenna is pointing. We have

$$0 \le G_T(\theta) \le 1, 0 \le G_R(\theta) \le 1 \quad \textbf{(1)}$$

We assume the generic two-ray propagation model:

$$P_R = K \cdot P_T \cdot G_T(\theta_T) \cdot G_R(\theta_R) / d^\alpha \quad \textbf{(2)}$$

where P_T is the transmit power at the antenna boresight; P_R is the receive power; θ_T (θ_R) is the angle between the line connecting the transmitter and receiver, and the boresight direction of the transmitter (receiver) antenna; *d* is the transmitter-receiver distance; $\alpha \geq 2$ is the path-loss exponent; and *K* is a normalization factor.

We further assume by proper beam steering, the transmitter and receiver of each link can point their antennae directly at each other so that the power is the highest along the line connecting them. Thus, for each link *i*, $G_{Ri}(0) = G_{Ti}(0) = 1$. In the following discussion, we only consider the pair-wise physical interferences between links without considering the additional interferences due to the underlying multi-access protocol.

Let T_k and R_k denote the transmitter and receiver of link *k*. For brevity, we will also use T_k and R_k to denote their positions. Consider the interference of link *j* on link *i*. Let $\theta_{ij} = \angle T_i R_i T_j, \phi_{ji} = \angle R_j T_j R_i$ (Fig. 1). Given an SIR threshold SIR_0, for reception at link *i* not to be interfered by another link *j*, we have

$$SIR = \frac{P_t G_{Ti}(0) G_{Ri}(0) / d_i^\alpha}{P_t G_{Ri}(\theta_{ij}) G_{Tj}(\phi_{ji}) / |T_j - R_i|^\alpha} = \{\frac{|T_j - R_i|}{G^*_{Ri}(\theta_{ij}) G^*_{Tj}(\phi_{ji}) d_i}\}^\alpha \ge SIR_o \quad \textbf{(3)}$$

Here $G^*(\theta) = G(\theta)^{1/\alpha}$, $0 \le G^*(\theta) \le 1$. Define the "guard zone" $\Delta = SIR_0^{1/\alpha} - 1 > 0$, then **(3)** can be written as

$$|T_j - R_i| \ge (1+\Delta) d_i \cdot G^*_{Ri}(\theta_{ij}) \cdot G^*_{Tj}(\phi_{ji}) \quad \textbf{(4)}$$

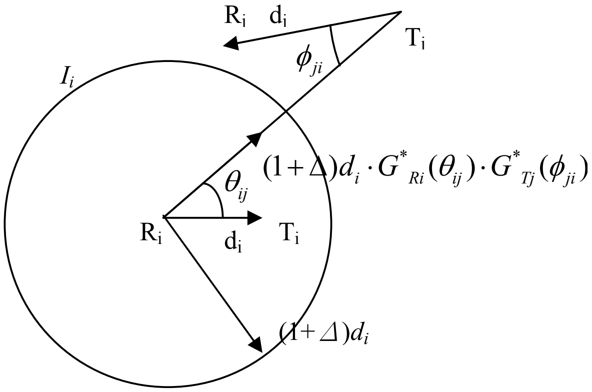

Fig.1. Directed interference from link *j* to link *i*

B. *Potential Interference Region*

Define the *potential interference region* of an active link *i* as a vulnerable area associated with R_i within which the transmission of T_j (*j*≠*i*) *may* interfere with the transmission from T_i to R_i. Particularly, the pair-wise potential interference region of link *i* is $I_i = \{x : |x - R_i| < (1+\Delta) d_i\}$, according to (4) and (1).

C. *Smart Antenna and Null Pattern*

By (4), if $G^*_{Ri}(\theta_{ij}) = 0$ or $G^*_{Tj}(\phi_{ji}) = 0$, the directed interference from link *j* to link *i* will vanish regardless of SIR_0. A null is required at the pattern of either the receiver or the interfering transmitter. A real-world implementation of such null pattern is smart antenna, formed by an array of antennae whose phases can be adjusted so as to control the beam direction and null direction. With *N* elements in an array, an array factor $AF(\theta) = |\sum_{i=0}^{N-1} a_i e^{-j\pi i \sin\theta}|$ is achievable [5], where $a_i (0 \le i \le N-1)$ can be arbitrary complex numbers. Note that the magnitude of the electric field is proportional to the array factor. Therefore the antenna power pattern $G_N(\theta) \propto AF_N(\theta)^2$ shares the same nulls. Let $z = \exp(-j\pi \sin\theta)$, $z_i = \exp(-j\pi \sin\theta_i)$. Given $z_1, z_2, \ldots, z_{N-1}$,

$$\prod_{i=1}^{N-1}(z - z_i) = \sum_{i=0}^{N-1} a_i z^i = \sum_{i=0}^{N-1} a_i e^{-j\pi i \sin\theta} \qquad \textbf{(5)}$$

From the above, we can calculate the corresponding $a_i(0 \le i \le N-1)$ required to position *N-1* Nulls or *N-2* Nulls plus the main beam angle arbitrarily. This nice property can be applied to solve the scalability problem of network capacity. Also, by filter design techniques in signal processing theory, given an arbitrary antenna power pattern $G(\theta)$, one can identify its best approximation using an antenna array with $G_N(\theta)$, under a specific criterion. With sufficiently large *N*, approximation error becomes asymptotically small. Thus, our analytical result of $G_N(\theta)$ can be a good approximation to that of an arbitrary antenna pattern $G(\theta)$.

III. SCALE LAW OF NETWORK CAPACITY WITH SMART ANTENNAE

As in [1], we consider the transport capacity for arbitrary networks and throughput capacity for random networks. In an arbitrary network, we are free to place nodes, choose source-destination pairs and transmission distance of each node. Transport capacity is the total bit-distance product per second that can be transported in the wireless network. Since the dimension of the area is bounded by the diameter of the disk, $D = 2\sqrt{A/\pi}$, if the channel capacity is *W*, an upper bound for the transport capacity is $TrC_u = 2W\sqrt{A/\pi}\lfloor n/2 \rfloor = \Theta(n)$ **(6)**

which can be achieved by a smart antenna with $N=\Theta(n)$ elements. A concrete example is shown in Fig. 2 where $n=10$ and $N=5$.

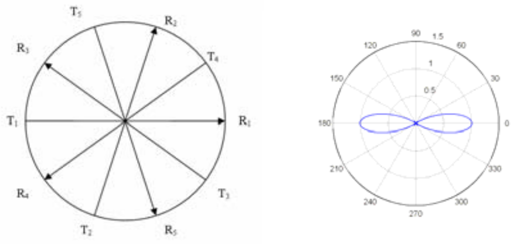

Fig.2. Node position, traffic pattern (left) and array factor (right)

In fact, for each odd number *N* and $n=2N$, we only need to select $a_i(0 \le i \le N-1)$ according to (5), where the nulls reside at $\theta_i = 2\pi i/N, i = 1,2,\ldots,N-1$. Nodes are placed on the circumference of the disk. The *i*th receiver is placed at azimuth $2\pi(i-1)/N, i = 1,2,\ldots,N-1$, while relative transmitter $2\pi(i-1)/N + \pi$. It is easy to verify that the transmission of each link will succeed without any interference induced by other links. The result of even *N* is similar. Therefore, with smart antennae, the transport capacity scales as $\Theta(n)$, although at the cost of $\Theta(n)$ elements of antenna array.

Also of interest are random networks, where the *n* nodes are independently placed in the disk in a uniformly random manner; and each source randomly and independently chooses its destination. All nodes share an identical transmission range, *r*, the maximal transmission distance. We consider the end-to-end throughput capacity. As indicated by the example above, it is obviously that if we choose *r* to be large enough to cover the entire disk, and equip each transmitter and receiver with $N = \Theta(n)$ elements of array, the upper bound of throughput capacity $ThC_u = W\lfloor n/2 \rfloor = \Theta(n)$ is achievable. What about *N* of lower order? For simplicity, the following analysis is on *average* and *asymptotic* basis. First, to guarantee connectivity, *r* should be ([1]): $r \ge c_1\sqrt{A \log n / n\pi}$ **(7)**

Note that the transmitter (or receiver) density is $n/2A$, the area of potential interference region of each link is $\pi(1+\Delta)^2 r^2$. The active interfering (interfered) neighbor of each receiver (transmitter) $k = n_{act}\pi(1+\Delta)^2 r^2/2A$, **(8)**

where n_{act} is the number of active nodes in the progress of transmission. Also note that on average, an end-to-end transmission distance is $c_2(A/\pi)^{1/2}$. Therefore there are $c_3(A/\pi)^{1/2}/r$ hops per end-to-end transmission on average. The throughput capacity is given by

$$ThC = n_{act}W/2/(c_3\sqrt{A/\pi}/r) = c_4 n_{act} Wr/\sqrt{A}\,, \qquad \textbf{(9)}$$

Finally, at each instant in time, there are $n_{act}k/2$ pairs of directed interference from active transmitters to active receivers, which can be eliminated by Nn_{act} nulls. Hence,

$$n_{act}k/2 \le Nn_{act},\ k \le 2N.\ n_{act} \le n. \qquad \textbf{(10)}$$

Substituting (10) and (8) in (9), we have

$$ThC = c_4 n_{act} Wr/\sqrt{A} \le cW\sqrt{Nn_{act}} \le cW\sqrt{Nn_{act}} \qquad \textbf{(11)}$$

However, the second equality holds when $n_{act}=n$. Therefore by (10), (7) and (8), $N \ge (1+\Delta)^2 c_1^2 \log n/2$. The number of nulls should exceed that of the interfering neighbors at the state of critical connectivity.

In the case where $N < (1+\Delta)^2 c_1^2 \log n/2$, we have $n_{act} \ne n$. Substituting (7), (8) and (10) in (9), we have

$$\begin{aligned} ThC &= c_4 n_{act} r^2 W/\sqrt{A}r \le c_4 k 2AW/\sqrt{A}c_1\sqrt{A\log n/n\pi} \\ &\le c' WN\sqrt{n/\log n} \end{aligned} \qquad \textbf{(12)}$$

To sum up,

$$\begin{aligned} ThC_u &= \Theta(\sqrt{Nn}), N \ge (1+\Delta)^2 c_1^2 \log n/2 \\ ThC_u &= \Theta(N\sqrt{n/\log n}), N < (1+\Delta)^2 c_1^2 \log n/2 \end{aligned} \qquad \textbf{(13)}$$

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