Cell Group Decoupling Analysis of a Dynamic Channel Assignment Strategy in Linear Microcellular Radio Systems

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Abstract— In this paper, we develop a simple but very accurate analytical model for a channel borrowing based dynamic channel assignment strategy in linear microcellular systems. Our approach is to decouple a particular cell together with its neighbors, i.e., those cells under its interference range, from the rest of the system for finding the blocking probability of that cell. We call this the cell group decoupling analysis. This analysis is applicable to both homogeneous and heterogeneous traffic distributions. We show that the effect of this decoupling causes the blocking probability so obtained to be an upper bound. The bound is found to be very tight when compared with simulation results. Besides, this analysis gives accurate results to boundary cells as well as inner cells, and is therefore quite different from the other approaches which neglect boundary effects.

I. INTRODUCTION

Efficient use of the limited radio spectrum is important in cellular mobile systems. At present, the demand of radio channels has already far exceeded the capacity in many metropolitan areas. A promising way to meet this demand is to use microcells [1-3]. Linear microcellular networks can be used, for example, to cover the traffic on a highway or a long street while planar microcells can be used to cover busy metropolitan areas. The decrease of cell size however exacerbates the spatial variability of mobile traffic, and makes frequency planning infeasible. An important feature of dynamic channel assignment (DCA) is that it can adapt to the spatial as well as temporal variations of traffic load [4]. DCA, therefore, will play a prominent role in microcellular systems.

There are many studies of DCA algorithms based on computer simulation [2,5-7]. An exception is an idealized strategy called "maximum packing" (MP) [4]. The MP strategy is not practical because it requires system-wide information. However, MP is analytically tractable and it provides a lower bound on the overall blocking probability for all other DCAs. The modeling of MP in any realistic size network would result in a state space too large for direct computation and again computer simulation is used for performance evaluation. Many approximate models were developed and this includes Raymond [8], Xu and Mirchandani [9], Bakry and Ackroyd [10] and Yue [11]. For cellular engineering using DCA schemes, a way to obtain a tight *upper bound* on blocking probabilities will be of great interest when exact solution is not available.

In this paper, we develop a simple but very accurate analytical model for a very powerful dynamic channel assignment strategy called the *Borrowing with Directional Channel Locking* (BDCL) in linear microcellular systems. Our approach is to decouple a particular cell together with its neighbors, i.e., those under its interference range, from the rest of the system for finding the blocking probability of that cell. We call our approach the cell group decoupling analysis. Similar clustering approaches were used in [12] and [13] for the Equivalent Erlang-B Approximation and the light traffic approximation of DCA analysis. Cell group decoupling analysis is applicable to both homogeneous and heterogeneous traffic distributions. We will show that the effect of this decoupling causes the blocking probability so obtained to be an upper bound. The bound is found to be very tight when compared with simulation results. Besides, this analysis gives accurate results to boundary cells as well as inner cells, and is therefore quite different from the other approaches which neglect boundary effects. Another use of the analysis is that a very efficient cell group decoupling simulation algorithm can be readily developed along the same line for very large size networks.

II. BORROWING WITH DIRECTIONAL CHANNEL LOCKING STRATEGY

In channel borrowing based DCA strategies, channels are allocated to each cell on a nominal basis. When a call request arrives and finds all nominal channels busy, a channel is borrowed from a neighboring cell if the borrowing will not violate the cochannel interference constraints. By incorporating channel reassignment (or intra-cell handoff), the channel borrowing based DCA strategies can give a superior performance over the fixed channel assignment even at overload conditions by keeping the channel reuse distance between cochannel cells a minimum. Among the different channel borrowing based DCAs, the BDCL [6] strategy gives the lowest blocking probability. Our analytical model is therefore developed based on this strategy.

Two important features of the BDCL strategy are channel ordering and immediate channel reallocation. Channel ordering means that all nominal channels are ordered such that the first channel has the highest priority to be assigned to the next local call, and the last channel is given the highest priority to be borrowed by the neighboring cells. Immediate channel reallocation means that a channel is reallocated whenever possible to keep the active local calls to be packed towards front end of the nominal channel list.
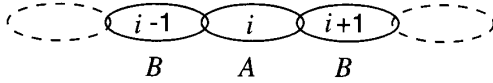
Fig. 1. A linear microcellular system with nominal channel sets A and B.

III. LINEAR MICROCELLULAR SYSTEMS

A. Problem Formulation

Consider a linear array of microcells shown in Fig. 1. Suppose there are a total $2m$ distinct frequency channels available to this microcellular system and the same frequency channel can be reused at every other cell. Then these channels can be divided into two nominal channel sets A and B with m channels in each set and consecutive cells can be allocated with channel sets $A, B, A, B, \ldots$ etc. Let S_i denote the set of active channels, or channels in use, in cell i and $|S_i|$ denote the number of elements in S_i. A call attempt arrives at cell i will be blocked if and only if $|S_{i-1} \cup S_i \cup S_{i+1}| = 2m$ because otherwise there will either be an idle channel in cell i or an idle channel can be borrowed from the neighboring cells to carry the call.

The two neighboring cells of cell i are cochannel cells using the same set of nominal channels. Therefore, it is easy to see that the following two statements about channel borrowing initiated by cell i are equivalent:

1. Cell i borrows a channel, say channel k, from cell $i-1$ and locks channel k in cell $i+1$.
2. Cell i borrows channel k from cell $i+1$ and locks channel k in cell $i-1$.

By making use of the property that the two neighboring cells of a cell are cochannel cells, we obtain the following proposition.

Proposition 1. If a cell has one or more channels lent to its neighboring cells, it cannot borrow channels from its neighboring cells to accommodate new calls. In other words, channel borrowing is not a chain reaction.

Proof: Without loss of generality, assume the channels in set A is numbered from 1 to m and that in set B is numbered from $m+1$ to $2m$. Let cell i lend k channels numbered $m-k+1$ to m to cell $i+1$. Then, $|S_{i+1}| = m+k$ and $|S_i| \leq m-k$. If a new call arrives at cell i and finds that $|S_i| = a < m-k$, the idle nominal channel $a+1$ is assigned to the new call. If, instead, it finds $|S_i| = m-k$, the call will have to be blocked because borrowing from cell $i+1$ is forbidden by the fact $|S_i \cup S_{i+1}| = 2m$ and borrowing from cell $i-1$ is equivalent to borrowing from cell $i+1$. **Q.E.D.**

To determine the blocking probability at cell i by applying the cell group decoupling analysis, we form the three consecutive cells $i-1$, i and $i+1$ into a group and assume this group to be decoupled from the rest of the network. In other words the blocking probability at cell i is assumed to depend only on its two neighboring cells. This assumption is justified by the non-propagative borrowing property of Proposition 1.
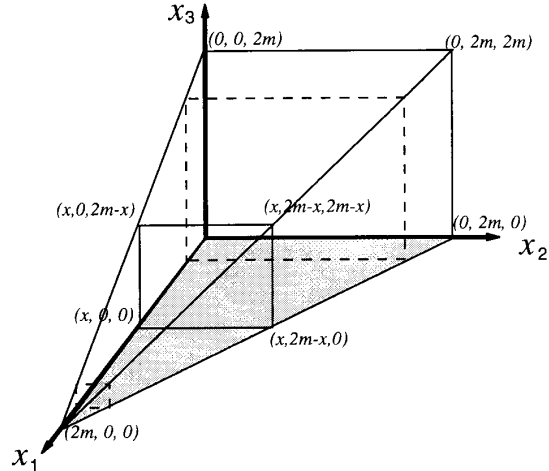

Fig. 2. A three-dimensional Markov chain formed by cells $i, i+1$ and $i-1$.

B. Calculation of Blocking Probability

Let the call arrivals at cell i be a Poisson process with rate λ_i and channel holding time be exponentially distributed with mean $1/\mu$. Let B_i be the call blocking probability at cell i. As a call in cell i is blocked if and only if $|S_{i-1} \cup S_i \cup S_{i+1}| = 2m$, we have

$$B_i = Prob\{|S_{i-1} \cup S_i \cup S_{i+1}| = 2m\}. \tag{1}$$

The channel usage in cells $i-1$, i and $i+1$ can be described by a three-dimensional ergodic Markov chain whose state space is shown in Fig. 2. This three-dimensional model is sufficient because with channel ordering and immediate channel reallocation, channels within a cell are indistinguishable and so the states of the three cell group can be described by the channel occupancies in the three cells. With (1) as the constraint, the total number of states is $\sum_{k=1}^{2m+1} k^2 = (8m^3 + 18m^2 + 13m + 3)/3$. Let $P(x_1, x_2, x_3)$ be the probability that $|S_i| = x_1, |S_{i+1}| = x_2$ and $|S_{i-1}| = x_3$.

Theorem 1. For a linear microcellular system with single cell buffering, the call blocking probability B_i of an arbitrary cell i obtained by the cell group decoupling analysis is an upper bound on the true blocking probability at cell i.

Proof: From (1), the channel occupancy S_i of cell i depends on both S_{i-1} and S_{i+1}. The maximum number of channels that can be used in cell i is therefore $2m - \max[|S_{i-1}|, |S_{i+1}|]$. In a decoupled three-cell system, if cell i has occupied $|S_i|$ channels, then the maximum number of channels that can be occupied by cells $i-1$ and $i+1$ is $2m - |S_i|$. On the other hand, if these three cells are not decoupled from the rest of the network, the channel occupancies of cells $i-1$ and $i+1$ are also affected by that of cells $i-2$ and $i+2$ respectively. As an example, if nominal channel k of cell $i+1$ is lent to cell $i+2$, then cell $i+1$ cannot use channel k. But cell i can use channel
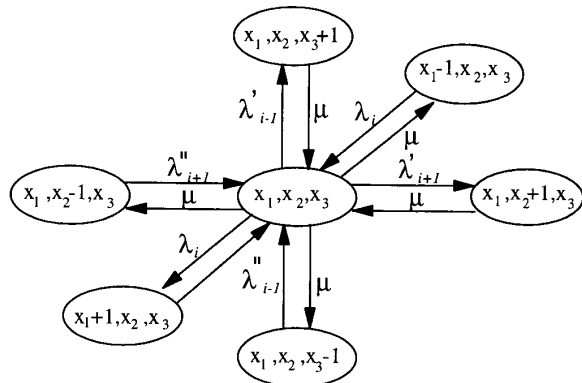
Fig. 3. A typical state transition diagram at state (x_1, x_2, x_3).

Fig. 4. Linear microcellular network with uniform traffic distribution.

k because cell i is the cochannel cell of cell $i + 2$. Thus in the non-decoupled model the blocking probability of cell i would be lower as more channels are available for use in certain states.

More formally, let $P_R(x_1, x_2, x_3)$ be the conditional probability that the total number of active channels in cells $i + 1$ and $i + 2$ is equal to $2m$ given that the current state is (x_1, x_2, x_3), or $P_R(x_1, x_2, x_3) = P\{|S_{i+1}| + |S_{i+2}| = 2m \,|(x_1, x_2, x_3)\}$. Let $P_L(x_1, x_2, x_3)$ be similarly defined to be the conditional probability that the total number of active channels in cells $i-1$ and $i-2$ is equal to $2m$ given that the current state is (x_1, x_2, x_3). Then the exact state transition rate from state (x_1, x_2, x_3) to (x_1, x_2+1, x_3) is $\lambda'_{i+1} = [1 - P_R(x_1, x_2, x_3)]\lambda_{i+1}$ and that from state (x_1, x_2, x_3) to $(x_1, x_2, x_3 + 1)$ is $\lambda'_{i-1} = [1 - P_L(x_1, x_2, x_3)]\lambda_{i-1}$ (Fig. 3). In the cell group decoupling analysis, we ignore the effect from cells outside the cell group and take the state transition rates simply as λ_{i-1} and λ_{i+1}. Since $\lambda_{i-1} > \lambda'_{i-1}$ and $\lambda_{i+1} > \lambda'_{i+1}$, we have in fact assumed a higher traffic rate and therefore the blocking probability obtained is larger, or is an upper bound of the exact blocking probability. By the same argument, λ''_{i-1} and λ''_{i+1} shown in Fig. 3 are similar quantities with values smaller than λ_{i-1} and λ_{i+1} respectively. **Q.E.D.**

The three-dimensional Markov chain shown in Fig. 2 happens to belong to the class of coordinate convex policy problems for which product-form solution can be obtained [14]. Let Ω denote the set of all possible states and $a_j = \lambda_j / \mu$. The steady state probability is given by

$$P(x_1, x_2, x_3) = G^{-1} \prod_{j=1}^{3} \frac{a_j^{x_j}}{x_j!} \qquad (2)$$

where

$$G = \sum_{(x_1, x_2, x_3) \in Omega} \left(\prod_{j=1}^{3} \frac{a_j^{x_j}}{x_j!} \right).$$

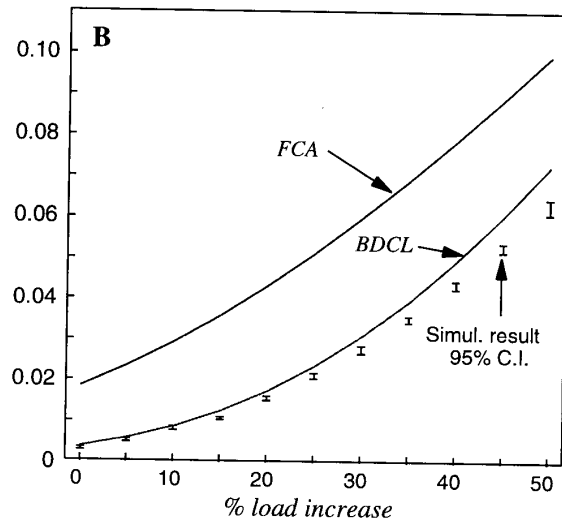From (1), the call blocking probability at cell i, is obtained

as

$$B_i = \sum_{x_1 + x_3 \le 2m} P(x_1, 2m - x_1, x_3) + \sum_{x_1 + x_2 < 2m} P(x_1, x_2, 2m - x_1). \qquad (3)$$

Note that a boundary cell has only one neighboring cell and so the three-dimensional Markov chain degenerates into a two-dimensional one. The overall blocking probability **B** of the system consisting of N linear microcells is given by

$$\mathbf{B} = \left(\sum_{k=1}^{N} \lambda_k \right)^{-1} \sum_{k=1}^{N} \lambda_k B_k. \qquad (4)$$

IV. ILLUSTRATIVE EXAMPLES

The linear microcellular system being analyzed consists of 30 microcells. Let there be a total of 20 channels in this system. Let the arrival of calls be a Poisson process and the call duration be exponentially distributed with a mean of 3 minutes.

First, we examine the performance under the homogeneous traffic distribution. Let the base load traffic in each cell be 100 calls/hour. The overall average blocking probability **B** of the system obtained by the above analysis is plotted as a function of the increase in traffic in Fig. 4. The Erlang-B formula which corresponds to the fixed channel assignment (FCA) performance is also plotted for the purpose of comparison. We see that under all traffic conditions, the blocking probability obtained by simulation is upper bounded by the analytical results. The bound tends to be looser at heavy traffic conditions. This is because the influence of the farther away neighboring cells on the blocking of the local cell grows with the traffic load and our analysis has ignored such influence.
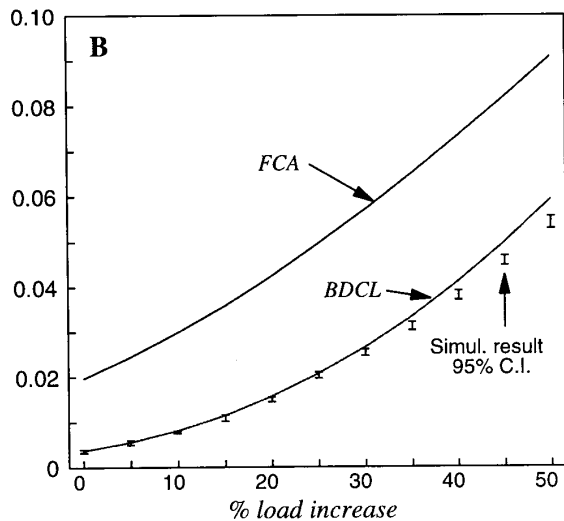
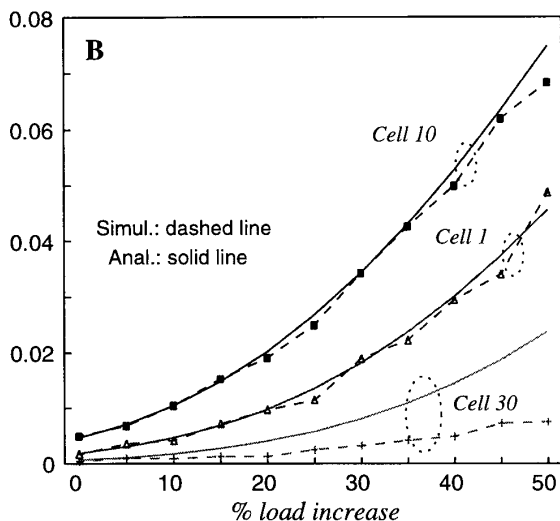Fig. 5. Linear microcellular network with nonuniform traffic distribution.

Fig. 6. Boundary cells of a linear network with non-uniform traffic.

Next, consider the same linear cellular systems under heterogeneous traffic where the traffic rate in calls per hour in the 30 cells are [120, 100, 80, 80, 90, 100, 90, 130, 120, 60, 40, 60, 80, 100, 80, 120, 100, 60, 60, 80,120, 100, 80, 60, 60, 80, 100, 100, 80, 120]. Fig. 5 shows again that a very tight bound is obtained. All blocking probabilities obtained under heterogeneous traffic conditions are weighted by the traffic rates λ_i's.

In Fig. 6, we study the performance of cell group decoupling analysis on boundary cells. Blocking probabilities of the two boundary cells (i.e. cells 1 and 30) and an arbitrarily chosen interior cell (cell 10) are plotted against traffic rates for the above non-uniform traffic distribution.

It is interesting to note that although cells 1 and 30 have

the same base load of 120 calls/hr, the cell group decoupling analysis provides a tighter upper bound on cell 1 than on cell 30. The reason is that for cell 30, the effect from the "neighbor's neighbor" (i.e. cell 28) is much stronger than that of cell 3 to cell 1. In other words, the base load of cell 28 is larger than the base load of cell 29 whereas the base load of cell 3 is smaller than the base load of cell 2. Cell 28, therefore, affects more on cell 29 than cell 3 on cell 2. In cell group decoupling analysis, we ignore the effect of "neighbor's neighbors" and so the bound tends to be looser when such effect is large at heavy load conditions.

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