Blocking and Handoff Performance Analysis of Directed Retry in Cellular Mobile Systems

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***Abstract*—A new analytical model for finding the call blocking probability of a cellular mobile system with directed retry is developed. It can give very accurate results for systems with both uniform and nonuniform traffic distributions. With that, we are able to formulate a second analytical model for obtaining the probability of additional handoffs due to directed retry. Numerical results show that the probability of additional handoff due to directed retry is more sensitive to the percentage of cell overlap than to the mean path length travelled by a mobile unit. In our example of a cellular system with a typical 30% cell overlap the probability of additional handoff is about 0.02. The use of directed retry, therefore, is expected to cause only a minimum amount of additional load in handoff processing and has only a minimal effect on the probability of handoff failure.**

I. Introduction

Efficient channel utilization is a major concern in cellular mobile systems over the years. Many channel assignment schemes with different utilization factors and complexities were proposed [1]–[4]. The fixed channel assignment (FCA) scheme is the most basic one and requires very simple control. Dynamic and hybrid channel assignment schemes have higher channel utilization but require more sophisticated control algorithms and hardwares. Those algorithms rely heavily on the channel usage information of remote cells and require a lot of inter-cell signaling.

Eklundh [5] proposed a variation of FCA called directed retry. Directed retry preserves the merits of the FCA and at the same time, reduces the blocking probability of a cell by increasing its channel utilization. It takes advantage of the inevitable overlap that exists among cells to allow some subscribers in a cellular mobile system to look for free radio channels in more than one cell. The improvement is accomplished at the expense of an increased number of handoffs and an increased level of cochannel interference.

In this paper, we first develop a new analytical model to study the call blocking probability of systems with directed retry. While a more precise model using the equivalent random method [8] to model the overflow traffic can be found in [5], it is not clear how it could be generalized to systems with nonuniform traffic rates among cells. The model we developed, however, is applicable to systems with nonuniform traffic distributions and is independent of the cell layout topologies. Very good agreement between analytical and simulation results are obtained. We then develop an analytical model to study the additional handoffs due to directed retry. Previous analytical studies on handoffs *without* directed retry can be found in [6]–[7].

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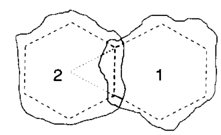


Fig. 1. Hypothetical radio coverage for a cellular system.

II. Directed Retry Strategy

The hexagonally shaped cell structure that is often used to represent a cellular radio coverage is, in practice, nonexistent. Due to the interference of the buildings and the variations of the terrain, the radio coverage is often of irregular shape. In order to design a system with full radio coverage over a certain area, at least 30% overlap of cells is needed. Fig. 1 shows a typical radio coverage of two cells, together with the nominal hexagonal partitioning. The shaded region is the overlapped radio coverage area. Mobiles in this region can hear from both cells 1 and 2 and therefore call requests can access channels from both cells.

In directed retry, a mobile unit in cell i initiates a call request on a common signaling channel to its base station. After the reception of the request, the base station will check to see if there is a free channel in cell i. If there is, the base station will assign a free channel to the calling mobile; if there is no free channel, the base station will provide the calling mobile with all its neighboring cells' common signaling channel identification numbers. The calling mobile will check the quality of the common signaling channels in these neighboring cells. If the quality of all of these common signaling channels is lower than the preset threshold value, the call is blocked; otherwise, the calling mobile will send a directed call-retry message through the qualified common signaling channel to request a channel in the new cell. If the base station of the new cell has a free channel, the directed call request is accepted in the new cell. Otherwise, it is rejected.

III. Blocking Performance of Directed Retry

A. Analytical Model

Consider a cellular system with a regular hexagonal cell structure. Let λ_i be the combined traffic rate of the new and handoff calls in cell i and let σ_i be the traffic rate in cell i due to directed retry. Let m_i be the number of channels in cell i and let f_i be the fraction of overlaid area in cell i (6) in next section shows how f_i can be calculated from the cell geometry). Also, let S_i be the set of neighboring cells of cell i and $|S_i|$ be the number of elements in set S_i. Assuming that the superposition of new calls, handoff calls and directed retry calls is a Poisson process and the duration of all calls is exponentially distributed with mean $1/\mu$, then the channel occupancy in cell i can be modeled by an M/M/m/m queue whose state transition rate diagram is shown in Fig. 2. The probability of finding all m_i channels busy is just the equilibrium probability of state m_i, denoted as b_i, or

$$b_i = \left[\sum_{k=0}^{m_i} \frac{(\rho_i)^k}{k!}\right]^{-1} \frac{(\rho_i)^{m_i}}{m_i!} \quad i = 1, 2, \cdots \tag{1}$$

where $\rho_i = (\lambda_i + \sigma_i)/\mu$. The arrival rate σ_i in cell i due to directed retry can be found by summing the contributions from all neighbors of cell i as follows

$$\begin{aligned}\sigma_i &= \sum_{j \in S_i} \lambda_j P\ [\text{cell}\, j \text{ is blocked and cell } i \text{ is not blocked}] \\ &\quad \cdot P[\text{a call is directed to cell } i |\, \text{cell}\, j \\ &\quad \text{is blocked and cell } i \text{ is not blocked}] \\ &= \sum_{j \in S_i} \lambda_j P\, [\text{cell}\, j \text{ is blocked}] \cdot P\, [\text{cell } i \text{ is not blocked}] \\ &\quad \cdot P[\text{a call is directed to cell } i |\, \text{cell}\, j \text{ is blocked} \\ &\quad \text{and cell } i \text{ is not blocked}] \\ &= \sum_{j \in S_i} \lambda_j b_j (1 - b_i) \frac{f_j}{|S_j|} \quad i = 1, 2, \cdots \end{aligned} \tag{2}$$

where we have assumed that the blocking probabilities of cells i and j are independent. It is obvious that when λ_i and f_i are not too large such that there are relatively little directed retry calls, the independence assumption is valid. Extensive numerical results shown in Figs. 4 and 5 reveal that the blocking probability in individual cells B_i obtained by analysis matches well with that of the simulation results even for B_i as large as 0.08. Returning, (1) and (2) can be solved simultaneously for the sets of unknowns $\{b_i\}$ and $\{\sigma_i\}$ with the initial values of all $\sigma_i = 0$. With that the blocking probability of a call initiated in cell i, denoted as B_i, can be obtained by conditioning on the events that the call is initiated in the overlaid and nonoverlaid areas of the cell as follows

$$B_i = (1 - f_i) b_i + \frac{f_i b_i}{|S_i|} \sum_{j \in S_i} b_j \tag{3}$$

($f_i/|S_i|$ is the probability that the call is initiated in one of the $|S_i|$ overlaid areas of cell i.) Therefore the overall blocking probability of a system consisting of N cells is given by

$$\mathbf{B} = \left(\sum_{k=1}^{N} \lambda_k\right)^{-1} \sum_{k=1}^{N} \lambda_k B_k. \tag{4}$$

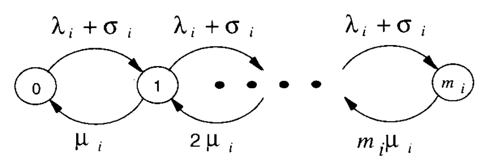


Fig. 2. The state transistion rate diagram of an M/M/m/m queue.

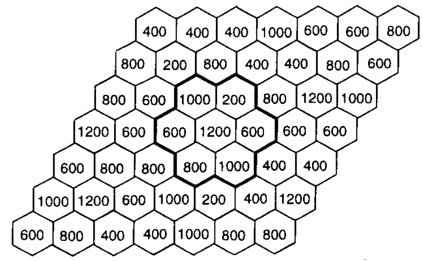


Fig. 3. Nonuniform traffic distribution in a 49-cell system.

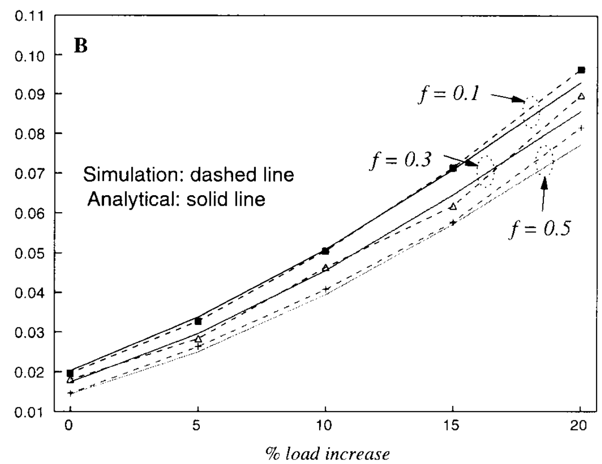


Fig. 4. Blocking performance under uniform traffic distribution.

B. Numerical Examples

To test the performance of the analytical model, we try it out on a 49-cell hypothetical network shown in Fig. 3. First consider a uniform traffic distribution among cells with base load of $\lambda_i = 1000$ calls/hour for all i. Also let $m_i = 60$ for all i and $1/\mu = 3$ minutes. Fig. 4 shows the blocking probability against the percentage increase of the traffic over the base load. Curves for $f_i = 0.1$, 0.3 and 0.5 are plotted. Good agreement with simulation results is observed. Next, consider a nonuniform traffic distribution where the call arrival rates to cells are shown inside each cell in Fig. 3. In Fig. 5, the call blocking probability is plotted against the percentage increase in traffic over the base load. Good agreement between analytical and simulation results is again obtained.

IV. Handoff Analysis for Directed Retry

A. Analytical Model

Accurate modeling of handoff activities is difficult because factors such as the irregular cell boundary, the layout of roads

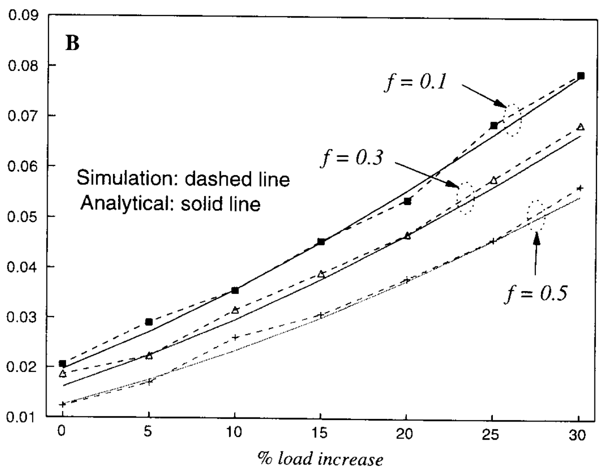


Fig. 5. Blocking performance under nonuniform traffic distribution.

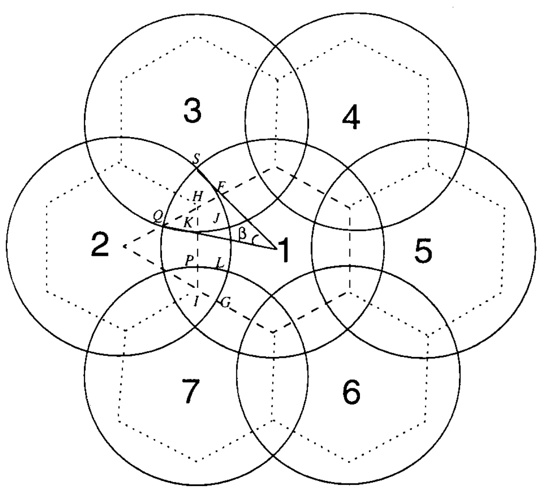


Fig. 6. Cell 1 and six neighboring cells.

in the cell, the traffic condition, the travel behavior of the subscribers, the calling behavior of different customer types, etc. all have to be accounted for. To study the additional handoff induced by directed retry, our model assumes the following:

1) The irregular radio coverage of a cell is approximated by a circle (Fig. 6) with radius R.
2) Mobile units initiating calls are uniformly distributed in a cell.
3) A mobile unit engaging a call moves in a straight line at bearing θ, where θ is uniformly distributed on $[0, 2\pi]$.
4) The path length U travelled by a mobile unit during a call is exponentially distributed with mean u.
5) The call arrivals at cell i is a Poisson process with rate λ_i and the call duration is exponentially distributed with mean μ^{-1}.
6) Cell i has m_i channels.

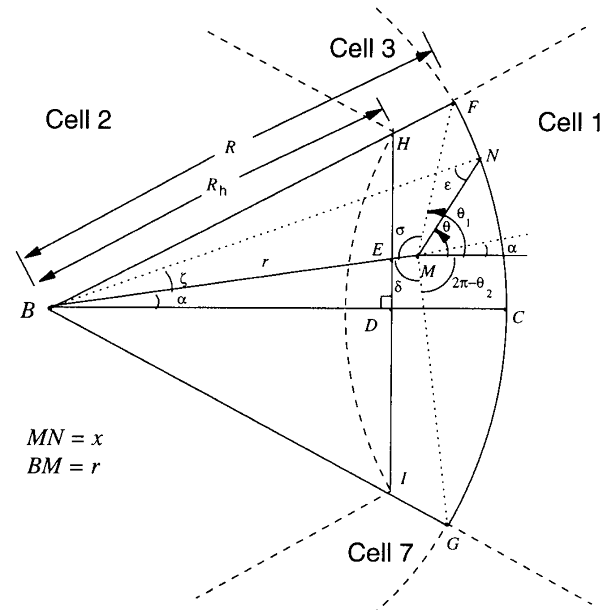


Fig. 7. Handoff analysis model.

The probability of additional handoff $\mathbf{P}_{AH}$ in a cell is caused by directed retry to all the neighbors of that cell. Let us first consider $P_{AH}^{(1,2)}$, the probability of additional handoff in cell 1 due to retry directed to a particular neighboring cell, say, cell 2. Fig. 7 shows a mobile unit M in the overlapped boundary between cells 1 and 2. Let M be assigned a channel from cell 2 by directed retry. When the mobile unit crosses arc FCG, an additional handoff is generated. To calculate the probability of additional handoff $P_{AH}^{(1,2)}$ in cell 1 due to retry directed to cell 2, we argue as follows:

$$\begin{aligned} P_{AH}^{(1,2)} =& P[\text{a call is initiated in the overlapped area } HFGI \text{ and the call is directed to cell 2}]\cdot \\ & P[\text{the mobile crosses the arc} FCG \text{ before call termination}] \\ =& p_1 \cdot p_2. \end{aligned} \tag{5}$$

Let the side length of a hexagonal cell be R_h. The overlapped area *HFGI* consists of three regions (Fig. 6): region 1 (area *HFJK*), region 2 (area *KJLP*) and region 3 (area *PLGI*). The probability p_1 can be obtained by conditioning on the location of the mobile unit in those three regions. Let $\Re_i$ denote the event that a call is initiated in region i. Since mobile units initiating calls are assumed to be uniformly distributed in the cell, $P[\Re_i]$ is numerically equal to the ratio of the area of region i to the area of a hexagonal cell. Let A_i be the area of region i. Then A_1 is equal to one third of the intersection area of the effective radio coverage *SJQ* and is given by

$$A_1 = \frac{R^2}{6}\left[2\sqrt{3}\left(\sin\frac{\beta}{2}\right)^2 + 3\beta - 3\sin\beta\right],$$

where

$$\beta = 2\cos^{-1}\left[\frac{\sqrt{3}R_h}{2R}\right] - \frac{\pi}{3}.$$

Due to symmetry, A_3 is equal to A_1. A_2 is obtained to be

$$A_2 = \text{area}\, HFGI - A_1 - A_3 = \frac{\pi R^2}{6} - \frac{\sqrt{3}R_h^2}{4} - 2A_1.$$

Thus we have

$$P[\Re_i] = \frac{A_i}{\text{area of a hexagonal cell}} = \frac{A_i}{(3\sqrt{3}/2)R_h^2}.$$

The probability f that a mobile unit can hear from other base stations is given by the ratio of the overlapped area in a cell to the area of the hexagonal cell. For cell 1 in Fig. 6,

$$f = \frac{6(A_1 + A_2)}{(3\sqrt{3}/2)R_h^2}. \quad (6)$$

For cells with less number of neighbors f can similarly be found.

Since both cells 2 and 3 can accommodate the directed calls from region 1 of cell 1, we assume the probabilities of choosing either cell to be equal. Let ξ denote the event a call is directed to cell 2. Then p_1 can be expressed as

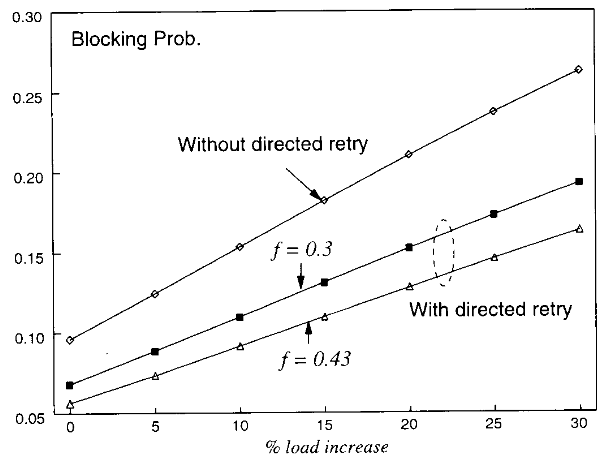
$$\begin{aligned} p_1 &= \sum_{i=1}^{3} P[\xi|\Re_i] \cdot P[\Re_i] \\ &= [b_1 b_3(1-b_2) + b_1(1-b_3)(1-b_2)/2]P[\Re_1] \\ &\quad + b_1(1-b_2)P[\Re_2] + [b_1 b_7(1-b_2) \\ &\quad + b_1(1-b_7)(1-b_2)/2]P[\Re_3]. \end{aligned}$$

where b_i is the probability that all channels in cell i are occupied and is given by (1). We refer the readers to the Appendix for the derivation of p_2. It involves tedious trigonometric manipulations but is conceptually simple.

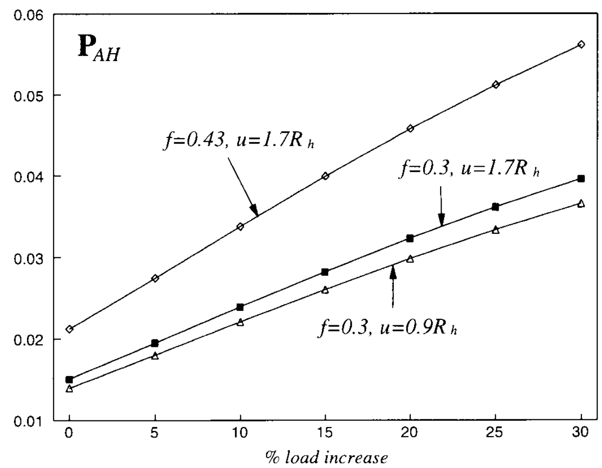
Having derived $P_{AH}^{(1,2)}$, we do the same for the other neighboring cells (Fig. 6) and combine their contributions to get $\mathbf{P}_{AH} = \sum_{j=2}^{7} P_{AH}^{(1,j)}$. In general, let S_i be the set of neighboring cells of cell i. Then the probability of additional handoff in cell i is $\mathbf{P}_{AH} = \sum_{j\in S_i} P_{AH}^{(1,j)}$, and the rate of additional handoff is just $\lambda_i \mathbf{P}_{AH}$.

B. Numerical Examples

Consider the *center* cell with base traffic load $\lambda_c = 1200$ calls/hr shown in Fig. 3. Fig. 8(a) shows the blocking probabilities with and without directed retry against the percentage increase of traffic over the base load (shown in Fig. 3. It is seen that at $f = 0.3$ (i.e., at 30% cell overlap) the blocking probability is reduced to about $\frac{2}{3}$ with the use of directed retry. At higher percentage of cell overlap, say $f = 0.43$, the blocking probability is reduced further to about $\frac{1}{2}$ of that without directed retry. Fig. 8(b) shows the probability of additional handoff in the center cell $\mathbf{P}_{AH}$ against the same percentage increase of traffic load (i.e., the same x-axis) with parameters ($f = 0.3$, $u = 0.9R_h$), ($f = 0.3$, $u = 1.7R_h$) and ($f = 0.43$, $u = 1.7R_h$). It can be seen that $\mathbf{P}_{AH}$ always increases with the traffic load. Thus combining Fig. 8(a) and



(a)



(b)

Fig. 8. (a) Blocking probability versus traffic load. (b) P_{AH} of the center cell versus traffic load.

(b), we can infer that $\mathbf{P}_{AH}$ increases almost linearly with the increase of blocking probability.

At a loading of 10% over the base load shown in Fig. 3, the probability of additional handoff in the center cell $\mathbf{P}_{AH}$ against u, the mean path length travelled by an active mobile unit is obtained and is shown in Fig. 9. The curve with $f = 0.3$ and at $u = 0.9R_h$, $\mathbf{P}_{AH} = 0.022$. $\mathbf{P}_{AH}$ in general is quite flat for $u > 0.6R_h$. Fig. 10 shows $\mathbf{P}_{AH}$ as a function of R for various values of u. We see that as R increases, so does the cell overlap ratio f [from (6)]. This in turn causes $\mathbf{P}_{AH}$ to increase. It is interesting to note that $\mathbf{P}_{AH}$ exhibits a nearly linear relation with R.

It was shown in [5] that directed retry can reduce the call blocking probability under the assumptions that handoff calls are treated as new calls and there is no channel reservation for handoff or new calls. Under the above assumptions, the probability of handoff failure is the same as the probability of call blocking and is also reduced with the use of directed retry. But we have shown that for individual calls the handoff rates

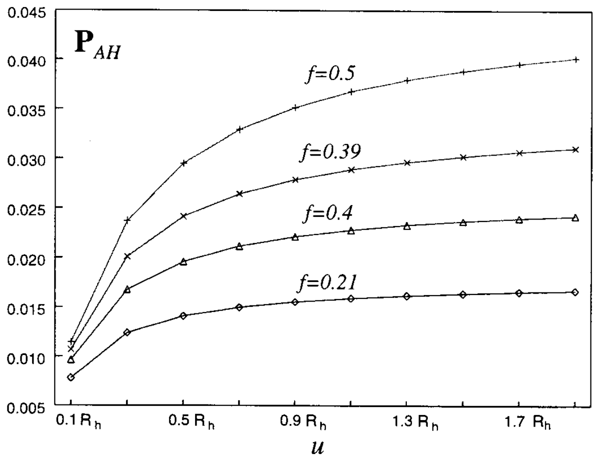


Fig. 9. Additional handoff probability versus mean path length.

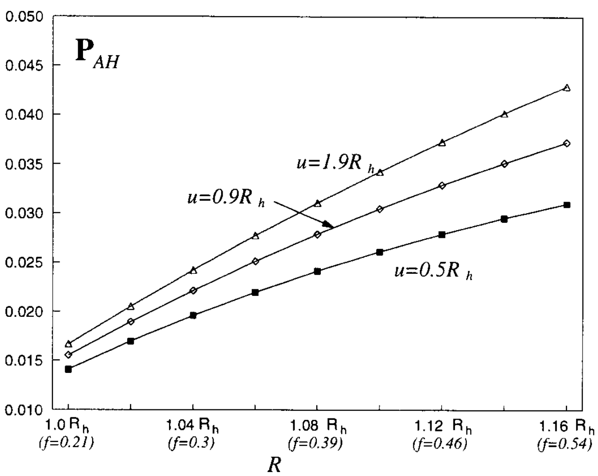


Fig. 10. Additional handoff probability versus effective radius of radio coverage.

(or the expected number of handoffs) is slightly increased. Therefore it is not clear whether the probability of handoff failure *for the duration of a call* is increased or decreased with the use of directed retry. Our numerical results under typical cellular operating conditions showed that the probability of requiring additional handoff is very small. Therefore we can safely conclude that directed retry has a minimal effect on the probability of handoff failure.

V. Conclusion

The blocking performance of directed retry under nonuniform traffic distribution and arbitrary topology is derived and is found to be in very good agreement with simulation. With that, we derive further the probability of additional handoff $\mathbf{P}_{AH}$ due to directed retry. We found that $\mathbf{P}_{AH}$ depends on the effective radio coverage radius R as well as the mean path length travelled by a mobile unit u. For a typical system with 30% cell overlap (or, $R = 1.04R_h$), the probability of additional handoffs due to directed retry is only about 0.022. The use of directed retry, therefore, would cause only a minimum amount of additional load in handoff processing and has only a minimal effect on the probability of handoff failure.

Appendix

To calculate p_2, consider mobile M at polar coordinate (r, α) and travelling in direction θ. Let $x(r, \alpha, \theta)$ be the distance between M and arc FCG at direction θ. Therefore, given r, α and θ, $p_2 = P[U > x(r, \alpha, \theta)]$. Removing the conditions on r, α and θ, we have

$$p_2 = 2\int_0^{\pi/6}\int_{r_0(\alpha)}^{R} \phi(r, \alpha)\frac{r}{\pi R^2/6 - \sqrt{3}R_h^2/4}\,dr\,d\alpha = \frac{12}{(2\pi R^2 - 3\sqrt{3}R_h^2)\pi}\int_0^{\pi/6}\int_{r_0(\alpha)}^{R} r\phi(r, \alpha)\,dr\,d\alpha.$$

where

$$\begin{aligned}\phi(r, \alpha) &= \int_0^{\theta_1(r,\alpha)} P[U > x(r, \alpha, \theta)]\cdot\frac{1}{2\pi}\,d\theta + \int_{2\pi-\theta_2(r,\alpha)}^{2\pi} P[U > x(r, \alpha, \theta)]\cdot\frac{1}{2\pi}\,d\theta \\ &= \int_0^{\theta_1(r,\alpha)} e^{-x(r,\alpha,\theta)/u}\,d\theta + \int_{2\pi-\theta_2(r,\alpha)}^{2\pi} e^{-x(r,\alpha,\theta)/u}\,d\theta.\end{aligned}$$

The lower limit $r_0(\alpha)$ is just the length of line BE and is given by

$$r_0(\alpha) = \frac{BD}{\cos\alpha} = \frac{R_h\sin(\pi/3)}{\cos\alpha} = \frac{\sqrt{3}R_h}{2\cos\alpha}.$$

The other two limits θ_1 and θ_2 can be readily obtained from Fig. 7 as

$$\theta_1 = \pi + \alpha - \sigma, \quad \theta_2 = \pi + \alpha + \delta \tag{7}$$

where σ and δ are derived using the sine law and the cosine law on ΔFBM and Δ MBG respectively and is given by

$$\begin{aligned}\sin\sigma &= \frac{R\sin(\pi/6-\alpha)}{FM} = \frac{R\sin(\pi/6-\alpha)}{\sqrt{R^2+r^2-2Rr\cos(\pi/6-\alpha)}} \\ \sin\delta &= \frac{R\sin(\pi/6+\alpha)}{MG} \\ &= \frac{R\sin(\pi/6+\alpha)}{\sqrt{R^2+r^2-2Rr\cos(\pi/6+\alpha)}}.\end{aligned} \tag{8}$$

Substitute (8) into (7), we have

$$\begin{aligned}\theta_1 &= \pi + \alpha - \sin^{-1}\left[\frac{R\sin(\pi/6-\alpha)}{\sqrt{R^2+r^2-2Rr\cos(\pi/6-\alpha)}}\right] \\ \theta_2 &= \pi + \alpha + \sin^{-1}\left[\frac{R\sin(\pi/6+\alpha)}{\sqrt{R^2+r^2-2Rr\cos(\pi/6+\alpha)}}\right].\end{aligned}$$

To calculate $x(r, \alpha, \theta)$, we use the cosine and sine laws on ΔNBM and obtain

$$x = \sqrt{R^2 + r^2 - 2Rr\cos\zeta}$$

where

$$\zeta = \begin{cases} \alpha - \theta - \sin^{-1}\left[\dfrac{r\sin(\pi - \alpha + \theta)}{R}\right] & \text{if } 0 \le \theta < \alpha \\ \theta - \alpha - \sin^{-1}\left[\dfrac{r\sin(\pi - \theta + \alpha)}{R}\right] & \text{if } \alpha < \theta \le \theta_1 \end{cases}$$

in the range $0 \le \theta \le \theta_1$ and

$$\zeta = 2\pi - \theta + \alpha - \sin^{-1}\left[\frac{r\sin(\theta - \pi - \alpha)}{R}\right]$$

in the range $\theta_2 \le \theta < 2\pi$.

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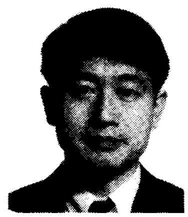

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