# The Nonuniform Compact Pattern Allocation Algorithm for Cellular Mobile Systems

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Abstract—An algorithm for allocating nominal channels according to traffic distribution is designed. The algorithm attempts to minimize the average blocking probability as nominal channels are allocated one at a time. Simulation results show that the system's traffic carrying capacity can be increased by about 10% with the use of this algorithm and that this gain is additional to the improvement obtained from the channel borrowing strategies. If the effect of shadow blocking is considered in the assignment of channels, only a very small increase in the traffic carrying capacity is observed.

#### I. Introduction

In recent years, there has been tremendous growth in cellular mobile communication in the developed countries [1]-[5]. As the frequency spectrum allocated for mobile telephone system is limited, congestion in some areas has already occurred. As a result, both the allocation of more bandwidth and the changing over to the more advanced digital technology have been planned. In fact as early as 1986, an additional bandwidth of 10 MHz was added to the existing U.S. system by the FCC to increase the total number of duplex channels to 832 [2].

Besides the above two methods, we want to show in this paper that with good channel management, the traffic-carrying capacity of the system could be increased significantly. There are two aspects of channel management: the allocation of nominal channels to the cells and the assignment of channels to serve the calls in each cell. Traditionally, the same number of nominal channels is allocated to each cell. This uniform allocation method is efficient if the traffic distribution of the mobile system is uniform. But since traffic distribution is more often nonuniform, some cells may have severe blocking while other cells have a sizable number of spare channels. One way to solve this problem is to use some form of channel borrowing strategy. Thus if a call arrives to a cell and all nominal channels in that cell are assigned, a channel from a neighboring cell is borrowed to serve the call. Such borrowing of channels, however, should not be overdone because for each channel being borrowed, the corresponding channels in three other cochannel cells are locked, affecting the throughput of these three cells. Thus a more effective solution is to allocate nominal channels to the cells in such a way that the average blocking in the entire system is minimized. Channel-borrowing strategies can then be applied to further reduce the call-blocking probability.

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In this paper, we shall first introduce a nominal channel allocation algorithm that allocates channels according to the traffic distribution of the system. We will then show by numerical examples that such allocation of channels can significantly increase the system's traffic-carrying capacity.

#### II. CHANNEL ALLOCATION

#### A. Compact Allocation Pattern

Let there be N cells and M channels in a cellular mobile system. If a channel is allocated to some cells without causing cochannel interference, these cells are called the cochannel cells of that channel. The allocation of a channel, say channel k, to the set of cochannel cells forms a pattern which we shall call the allocation pattern of channel k and denote as  $\pi_k$ .  $\pi_k$  is described by a set of indicator functions  $\{I_1(k), I_2(k), \cdots, I_N(k)\}$  where

$$I_i(k) = \begin{cases} 1, & \text{channel } k \text{ is allocated to cell } i \\ 0, & \text{otherwise.} \end{cases}$$
 (1)

Let the compact allocation pattern of a channel be the pattern with minimum average distance between cochannel cells. Fig. 1 shows two compact allocation patterns assuming the minimum cochannel reuse distance is three cell units. With the center cell moved to one of the neighboring cells, six other patterns are possible. The total number of compact patterns, therefore, is  $2 \times 7 = 14$ . We denote the set of compact patterns as  $G = \{g_1, g_2, \dots, g_{14}\}$ 

## B. Average Call Blocking

Let  $n_i(m)$  be the total number of channels allocated to cell i given that m channels are allocated to the system. Then

$$n_i(m) = \sum_{k=1}^m I_i(k), \qquad i = 1, 2, \dots, N.$$
 (2)

Let  $\lambda_i$  be the traffic in erlangs to cell i and let the number of channels available in the cell be  $n_i$ , then the call blocking probability in the cell is given by the Erlang B formula as

$$P(\lambda_i, n_i) = \left[\sum_{k=1}^{n_1} \frac{\lambda_i^k}{k!}\right]^{-1} \frac{\lambda_i^{n_i}}{n_i!}.$$
 (3)

The overall average blocking probability in the cellular mobile system is given by

$$PB(m) = \sum_{i=1}^{N} w_i p(\lambda_i, n_i(m))$$
 (4)

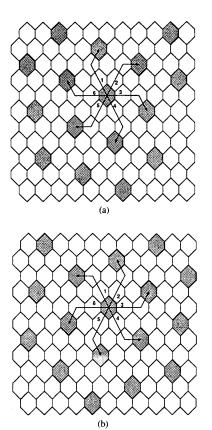


Fig. 1. Compact allocations patterns. (a) Clockwise pattern. (b) Anticlockwise pattern.

where

$$w_i = \frac{\lambda_i}{\sum_{i=1}^N \lambda_i}$$

is the traffic weighting factor.

#### C. Channel Allocation Algorithm

Disregarding all the constrains in the allocation of channels to cells, the total number of patterns for the allocation of one channel is  $2^N$  for an N-cell system. If M channels are to be allocated, the total number x is  $2^{NM}$ . Typically, for N=50 and M=70,  $x \approx 10^{1050}$ . Even if all the infeasible choices are eliminated, the resulting number is still astronomical. In this section, we will develop a heuristic algorithm for searching the optimal allocation pattern.

Let the allocation pattern for a channel that results in the largest drop of blocking probability be denoted as  $\pi^*$ . Consider the allocation of the first channel in the cellular system. The optimal allocation pattern for channel 1,  $\pi_1^*$ , is the pattern that minimizes

$$PB(1) = \sum_{i=1}^{N} w_i p(\lambda_i, n_i(1)).$$

As we have mentioned before, there are  $2^N$  possible patterns. In order to reduce the amount of search efforts, we

assume the optimal allocation pattern is of the compact type. That is  $\pi_1^* \epsilon G$ . Next, let the set of allocation patterns of the first m channels be  $\Gamma_m = \{\pi_1^*, \pi_2^*, \cdots, \pi_m^*\}$ . Then, to allocate the (m+1)st channel, we choose the allocation pattern that minimizes the blocking probability without changing the allocation of the previous channels. In doing so, the pattern search process is greatly simplified as no backtracking is necessary. With the addition of the (m+1)st channel, the number of allocated channels in cell i

$$n_i(m+1) = n_i(m) + I_i(m+1), \qquad i = 1, 2, \dots, N.$$
 (5)

So the average blocking probability after the allocation of the (m + 1)th channel is

$$PB(m+1) = \sum_{i=1}^{N} w_i p(\lambda_i, n_i(m) + I_i(m+1)).$$
 (6)

To minimize PB(m+1) with respect to  $\pi$ , we choose as usual the allocation patterns from G and denote it as  $\pi_{m+1}^*$ . With that, we have

$$\Gamma_{m+1} = \Gamma_m \cup \left\{ \pi_{m+1}^* \right\}. \tag{7}$$

Thus the allocation of channels on an M channel system can be performed successively starting from m=1. As a check, if the traffic in the system is uniform, the allocation algorithm indeed gives a uniform allocation of nominal channels to the system.

#### III. CHANNEL ASSIGNMENT STRATEGIES

After the nominal channels are allocated to the cells, several channel assignment strategies can be used to serve the calls in the cells [6]-[11].

#### A. Fixed Assignment Strategy

In fixed assignment (FA) strategy [4], [7], [9], the set of nominal channels is permanently allocated to each cell. In the FA strategy an arriving call can only be served by the nominally allocated channels. If all nominal channels are assigned, new calls are blocked. Most of the other strategies are variations of the FA strategy adopting different channel borrowing methods.

#### B. Borrowing with Channel Ordering (BCO) Strategy

In borrowing with channel ordering (BCO) strategy [8], all nominal channels are ordered such that the first channel has the highest priority to be assigned to the next local call and the last channel is given the highest priority to be borrowed by the neighboring cells. In the BCO strategy, after a channel is borrowed, it is locked in the cochannel cells within the channel reuse distance of the borrowing cell.

## C. Borrowing with Directional Channel Locking (BDCL) Strategy

In the borrowing with directional channel locking (BDCL) strategy [9], all nominal channels are ordered. When a channel is borrowed, the locking of this channel in the cochannel cells is restricted only to those affected by this

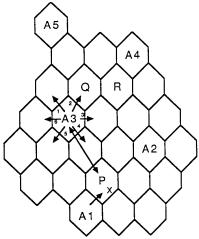


Fig. 2. Channel borrowing and directional locking.

borrowing. Consider Fig. 2 and let cell P borrow channel x from cell AI. Then channel x in cell A3 needs to be locked in directions 3, 4, and 5 only. Cells in directions 1, 2, and 6 are free to borrow channel x since their borrowing will not interfere with the call in cell P.

## D. Locally Optimized Dynamic Assignment (LODA) Strategy

In the locally optimized dynamic assignment (LODA) strategy, no nominal channel is assigned to the cells. Channels are shared by the entire mobile system. A particular cell having a call to serve evaluates the cost of using each candidate channel. The channel with the minimum cost is then assigned. The cost here is a measure of the future call blocking probability. Details of how the cost function is established and how the minimum cost channel is identified are given in [9].

# IV. CHANNEL ASSIGNMENT STRATEGY WITH SHADOW BLOCKING FACTOR<sup>1</sup>

In a cellular mobile system, there may be natural obstacles between two adjacent cells, that allows them to use the same channel without interference. As shown in Fig. 2, if the channel reuse distance between cell P and cell A3 is two cell units with shadow blocking (in contrast to a distance of three cell units without shadow blocking), then cell P and cell A3 can use the same channel simultaneously.

Although we may allocate a nominal channel to P and A3, the channel reuse efficiency cannot be increased because to guarantee the distance of all the other cochannel cells to be at least three cells apart, the distance between some adjacent cochannel cells has to be greater than three cell units. In other words, the total number of cells allocated with that channel remains the same.

The shadow blocking factor, however, does give some advantage in channel assignments. To investigate this, we

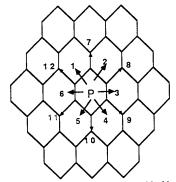


Fig. 3. The 12 directions of shadow blocking.

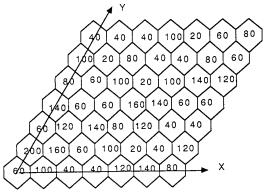


Fig. 4. Cellular system with nonuniform traffic distribution.

first construct a matrix to store the shadow blocking information for each cell. Considering cell P in Fig. 3, twelve directions regarding the shadow blocking factors (SBF) are indicated on P. If a channel can be reused in a neighbor cell, the SBF in that direction is set to 1. If a channel can be reused in a cell two cell units apart from P, SBF in that direction is set to 2. If there is no shadow blocking, SBF = 3. For cell P in Fig. 2, the value of SBF's in all directions except direction 1 is 3. In direction 1 of cell P, SBF(1) = 2. Cell A3 lies in direction 1 of cell P. Therefore, when cell P borrows a channel from A1, the channel is locked in the appropriate directions in A3 to prevent the channel from being borrowed in the locked directions. But with shadow blocking, A3 can use that channel to serve calls.

## V. PERFORMANCE EVALUATION VIA SIMULATION

The simulated cellular system has 49 hexagonal cells as shown in Fig. 4. Two integer variables x and y ( $1 \le x$ ,  $y \le 7$ ) are used to identify the cell locations. The numbers represent the call arrival rates in the respective cells. They range from 20 calls/hour to 200 calls/h. Fig. 5 shows an example of shadow blocking in the mobile system. Any two cells connected by a bidirectional arrow can use the same channel without interference. We assume that a total of 70 channels are available for the system. The arrival of calls is a Poisson process and the call duration is exponentially distributed with a mean of three minutes.

<sup>&</sup>lt;sup>1</sup> This study of the shadow blocking factor was suggested by Dr. Justin Chang of Bell Communications Research, Inc.

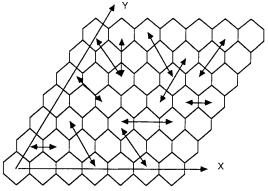


Fig. 5. Shadow blocking distribution.

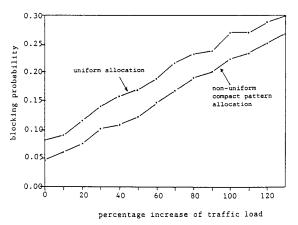


Fig. 6. FA assignment with nonuniform compact pattern allocation and uniform channel allocation.

Under the condition that the channel reuse distance is three cell units, Fig. 6 shows the average call-blocking probability of the FA strategy as a function of traffic load. The base traffic load is shown in Fig. 4. This traffic load is then increased by  $10\%, 20\%, \cdots, 130\%$  over the base load. We find that the blocking probability using the nonuniform compact pattern allocation is always lower than that using the uniform channel allocation, where each cell is allocated with 10 nominal channels. It is interesting to note that the reduction of blocking probability for this system is almost uniformly 0.04 for the range of traffic load shown. Also from Fig. 6, we see that at the same blocking probability of 0.08, the mobile system can carry 22% more traffic with the use of the nonuniform compact pattern allocation.

Similarly, Fig. 7 shows that at a blocking requirement of 3%, the BCO strategy with nonuniform allocation can carry 10% more traffic than that with uniform channel allocation. For the BDCL strategy at a blocking requirement of 2%, 10% more traffic carrying capacity with the use of nonuniform compact pattern allocation is observed (Fig. 8). If the shadow blockings, as shown in Fig. 5, is considered, an additional 2% more traffic carrying capacity is observed.

Finally, in Fig. 9, we compare the relative performance of FA, LODA, BCO, and BDCL strategies using the nonuni-

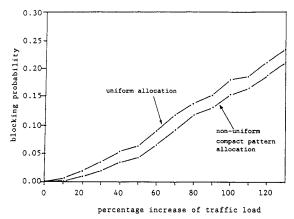


Fig. 7. BCO assignment with nonuniform compact pattern allocation and uniform channel allocation.

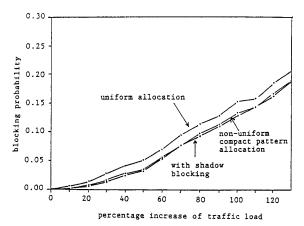


Fig. 8. BDCL assignment with nonuniform compact pattern allocation and uniform channel allocation.

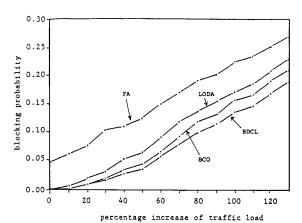


Fig. 9. Different channel assignment strategies with nonuniform compact pattern allocation.

form compact pattern. Compared with the results in [9], we see that the relative performance of the four assignment strategies remains the same. The performance gained from the nonuniform compact pattern allocation is additive to those gained from the borrowing strategies.

#### V. Conclusion

Our study shows that the allocation of nominal channels should be based on the actual traffic distribution in the cellular mobile system. With a good allocation strategy, our simulation result shows that the traffic carrying capacity can be increased by about 10%. This advantage, moreover, is additive to those gained from the channel borrowing strategies. As the traffic in a real system varies from hour to hour, dynamic updating of channel allocation is needed to maintain the system in optimum condition. Also, for systems with different cell size, a generalization of the allocation algorithm is needed.

We suspect that the ultimate capacity improvement obtainable from manipulating the allocation of nominal channels is only of the order of 10%. To cope with the future growth of traffic, going to digital cellular and increasing bandwidth allocation are inevitable. What we have proposed, however, is equally applicable to digital cellular systems.

#### REFERENCES

- D. C. Cox. "Portable digital radio communications—An approach to tetherless access," *IEEE Communications Mag.*, vol. 27, July 1989.
- [2] A. C. Stocker, "Small-cell mobile phone system," *IEEE Trans. Veh. Technol.*, vol. VT-33, Nov. 1984.
- [3] W. C. Y. Lee, Mobile Cellular Telecommunications Systems, New York: McGraw-Hill, 1989, p. 6.
- [4] V. H. MacDonald, "The cellular concept," Bell Syst. Tech. J., vol. 58, no. 1, Jan. 1979.
- [5] W. R. Young, "Advanced mobile phone service: Introduction, back-ground, and objectives," Bell Syst. Tech. J., vol. 58, no. 1, Jan. 1970
- [6] J. S. Engel and M. M. Peritsky, "Statistically optimum dynamic server assignment in systems with interfering servers," *IEEE Trans. Veh. Technol.*, vol. VT-22, Nov. 1973.
- [7] T. J. Kahwa and N. D. Georganas, "A hybrid channel assignment scheme in large scale, cellular structured mobile communication systems," *IEEE Trans. Commun.*, vol. COM-26, Apr. 1978.
- [8] S. M. Elnoubi, R. Singh, and S. C. Gupta, "A new frequency

- channel assignment algorithm in high capacity mobile communication systems," *IEEE Trans. Veh. Technol.*, vol. VT-31, Aug. 1982.
- [9] M. Zhang and T. S. Yum, "Comparisons of channel assignment strategies in cellular mobile telephone systems," *IEEE Trans. Veh. Technol.*, vol. 38, pp. 211-215, Nov. 1989.
- [10] D. C. Cox and D. O. Reudink, "Dynamic channel assignment in two-dimentional large scale mobile radio systems," *Bell Syst. Tech.* J., vol. 51, no. 7, Sept. 1972.
- 11] D. C. Cox, D. O. Reudink, "A comparison of some channel assignment strategies in large-scale mobile communications systems," *IEEE Trans. Commun.*, vol. COM-20, Apr. 1972.



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