# Networks of Open Interaction 

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#### Abstract

A network of open interaction (NOI) is an open access platform that enables agents distributed over a wide geographical area to remotely exert actions mutually or to an ambient environment. A valuable benefit of an open architecture is to allow open access so that it can be used to support a variety of individual or group objectives specified by the agents. In this paper we present some examples of NOI to serve as illustration. While much research on design techniques and analysis methodologies still awaits the attention of researchers, some germs of ideas relevant to NOI are presented here for consideration. The two main concepts are distributed control allowing choices and real-time medium access control built on protocol sequences.


Key Words: Information-based control, distributed control, real-time medium access control, protocol sequence

## 1 INTRODUCTION

While the Internet revolution is still raging on and bringing about monumental changes to our society, innovators and researchers are already exploring even more encompassing offshoots, such as the Internet of Things, Web of Things, Sensor Webs, Supervisory Control and Data Acquisition (SCADA), and Distributed Control Systems (DCSs), to name a few. In addition allowing people to disseminate and access information around the world at their fingertips, these potent scions engender the capability to act on remote objects. Speaking in hyperbole, the goal post of Internet has been advanced from omniscient to some limited degree of omnipotent.
Industrial control networks started out with closed architectures and gradually migrated towards the open architecture paradigm promoted by the OSI reference model. An open architecture allows the possibility of open access for agents who may be distributed all over the world. This openness in access is one of the key success factors for Internet and may prove to be as important for the development of control networks. The shift to openness is not just limited to accessibility, it is also reflected in the functionality of a network - agents can now define their individual or group objectives dynamically on the network. In traditional distributed control networks, goals are usually explicitly defined along with key system parameters. Whether the agents are cooperating or competing they are expected to know the intended target and act and react accordingly. For an open network, the set of active users can vary dynamically and even their identity may be unknown to each other. The exact task goal cannot be fully known to all and may change from time to time. In a human society, such ambiguity in people-to-people interaction is prevalent, although the sphere of influence of most individuals tends to be local. Advances in technology now allows the sphere of influence of individuals to be extended, even globally. Networks of interaction are the results of extrapolation of these development trends.

The word "interaction" emphasizes the fact that as agents exert controls to the network and observe the resulting effects, their information on the network goals are refined so

[^0]that future controls can be updated and optimized. In [1], the authors studied the interaction between two dancing partners. The model intermingles control, communication and observation into a non-classical, non-separation complex. A network of open interaction (NOI) extends this interaction complex to a network platform, so that one can envisage two dancing partners located in two separate continents can perform a virtual dance for the appreciation of the audience spread over the world.

## 2 EXAMPLES OF NOI

We illustrate the notion of a network of interaction through three examples.

## 1. Smart Grid

Consider a smart grid composed of a large number of independent generators running on renewable energy. The source nodes are labeled from 1 to $K$, while the load nodes (sink nodes) are labeled from $K$ to $L$. Using the model proposed in [19] and [16], the evolution of the network is assumed to be described as:

$$
\begin{gather*}
M_{i} \ddot{\theta}_{i}+D_{i} \dot{\theta}_{i}+\sum_{j \in N_{i}} V_{i} V_{j} b_{i j} \sin \left(\theta_{i}-\theta_{j}\right)=P_{i} \\
i=1, \ldots, L \tag{2.1}
\end{gather*}
$$

where $P_{i}$ represents the power input or output at node $i,\left(P_{i}\right.$ 's are expected to be positive for $i=1, \ldots, K$ and negative for $i=K+1, \ldots, L,) V_{i}$ represents the nodal voltage in rms value, $\theta_{i}$ is the voltage phase, $b_{i j}$ is the admittance of the line joining node $i$ and node $j, M_{i}$ is the motor's inertia constant, $D_{i}$ is the damping coefficient, and $N_{i}$ is the set of nodes connected to node $i$.

The parameters, $\left(V_{i}, P_{i}\right)$ for $i=1, \ldots, K$, can be regarded as control inputs although in some papers the $V_{i}$ 's are fixed and the model is reduced to the so called Kuramoto-like form (see [16].) A large amount of work has been devoted to analyzing the stability region and the dynamics of such a model. There is also a rich collection of problems that can be examined from the perspective of an NOI.

To give one example, consider the problem of coordinating the source nodes to provide power supply to satisfy the demands of the sink nodes. To be more specific, assuming that for $i=K+1, \ldots, L$, sink node $i$ has a publicized power demand curve given by the non-negative function, $R_{i}(t)$, for the period $\left[t_{0}, t_{1}\right]$. Assume also that each source node $i$ has
committed to supply $S$ units of power to the network during the same time period, where $S$ is defined by:

$$
\begin{equation*}
S=\int_{t_{0}}^{t_{1}} P_{i}(r) d r \tag{2.2}
\end{equation*}
$$

The exact shape of the function $P_{i}(r)$ is up to source node $i$ to decide individually. However, to ensure that the demands at all sink loads are met at each time instant, there is a preagreed penalty for under supply, so that the payment to each source supplier is of the form:

$$
\begin{equation*}
G=C_{0} S-C_{1} \sum_{j=K+1}^{L} \int_{t_{0}}^{t_{1}}\left(R_{j}(t)+P_{j}(t)\right)^{+} d t, \tag{2.3}
\end{equation*}
$$

for some constants $C_{0}$ and $C_{1}$. Here $x^{+}$is defined as $\max (x, 0)$.

If the source nodes are all coordinated by a central scheduler, this is a standard power scheduling and transmission problem. However, in the absence of a central scheduler, the problem becomes very complicated. Communication through the network is an important solution tool as there may not be any sideline communication. As in the dancing analogy, control and communication are mingled in an intricate manner for this type of problems.

## 2. Minority Games

Tremendous attention have been devoted to gain analytical understanding of the financial markets, involving researchers not only in the financial and economic areas, but also those from mathematics, physics and engineering. A real world model of any financial market is beyond the scope of this paper. However, it is noted that in recent years, researchers working in the econophysics area has proposed some relatively simple models that nevertheless captured salient aspects of the market dynamics. The Minority Game (MG) model is one such model, (see for example [7], [5] and [8].) Since these models have been set up to analyze the behavior of market traders, it is natural to consider them as examples of NOI. Here, we briefly describe one MG based problem that has control theoretic context.

Consider an MG model with $N$ trading agents (individual investors.) Each agent at time step $t$ can decide either to "buy" or "sell". These actions are represented by the integer, 1 , and -1 respectively. Let $a_{i}(t)$ represent the action of the $i$ th agent at time step $t$. Since a buyer-dominant decision would drive up the price benefiting the minority sellers, the gain to agent $i$ is given by [7]:

$$
\begin{equation*}
g_{i}(t)=-a_{i}(t) A(t) \tag{2.4}
\end{equation*}
$$

where $A(t)$ is sum of the decisions of the agents at step $t$ :

$$
\begin{equation*}
A(t)=\sum_{i=1}^{N} a_{i}(t) \tag{2.5}
\end{equation*}
$$

An agent's decision is determined by a deterministic strategy which is based on the market history, defined in the following way. Let

$$
h(t)= \begin{cases}1, & \text { if } A(t) \geq 0  \tag{2.6}\\ -1, & \text { otherwise }\end{cases}
$$

Let integer $M$ represent the depth of the historical memory, and define the binary market history sequence before time $t$ for $t \geq M$ by

$$
\begin{equation*}
\mathbf{h}(t)=(h(t-M), h(t-M+1), \ldots, h(t-1)) . \tag{2.7}
\end{equation*}
$$

Agent decisions at step $t$ are determined based on these sequences according to the strategy selected. In other words, each strategy, $S$, can be regarded as a function from $\mathbb{Z}_{2}^{M}$ to $\mathbb{Z}_{2}$ so that the strategy $S$ defines the action at $t$ to be $s(\mathbf{h}(\mathbf{t}))$. The strategy space, $\mathcal{S}$, has a size of $2^{2^{M}}$ which is extremely large if the depth of memory $M$ is long. There is a rich set of literature investigating how to reduce the complexity of the strategy space and to understand the effect of various strategies on the long term performance of the market; (see for example [14].)

We can reinterpret MG models as NOI by highlighting the estimation-control elements. As an example, consider a special situation where the strategies are restricted to a positive strategy, $p$, or a negative strategy, $n$, only. These strategies are defined as:

$$
\begin{equation*}
p(\mathbf{d})=\operatorname{sgn}\left(\sum_{i=1}^{M} d_{i}\right), \quad n(\mathbf{d})=-p(\mathbf{d}) \tag{2.8}
\end{equation*}
$$

where $\mathbf{d}$ is an element of $\mathbb{Z}_{2}^{M}$ and $\operatorname{sgn}(0)$ is interpreted as 1 .
At step 0 to $M-1$, agents are assumed to derive their decisions randomly and at step $M$, each agent is randomly assigned a strategy of either $p$ or $n$. For step $t$ greater than $M$, the strategy used by the $i$ th agent, denoted by $S_{i}(t)$, evolves according to the rules:

$$
\begin{align*}
& S_{i}(t)=\operatorname{sgn}\left(c_{i}(t)\right) p, \\
& c_{i}(t)=\left\{\begin{aligned}
c_{i}(t-1)+\beta_{i} g_{i}(t-1) & \text { if } S_{i}(t-1)=p, \\
c_{i}(t-1)-\gamma_{i} g_{i}(t-1) & \text { if } S_{i}(t-1)=n,
\end{aligned}\right. \\
& c_{i}(M)=1 \text { or }-1, \text { depending on the strategy } \\
& \text { assigned at step } M . \tag{2.9}
\end{align*}
$$

Here $\beta_{i}$ and $\gamma_{i}$ are individual scaling parameters.
These update rules can be understood by interpreting $c_{i}$ as a confidence function of the effectiveness of the positive strategy. An agent who has received positive gain from the currently used strategy will become more convince of it or lose confidence in it otherwise. A change of strategy takes place when there is a change in sign for the confidence function.

So far we have described the basic dynamics of the model. In order to introduce the control element, consider the scenario where a subset of the agents may form groups, each of which is controlled by a group leader. Moreover, all group leaders have access to market projection given in the form of $\rho(t)$, the probability distribution of the sum of the decisions of the individual (non-group) investors at step $t$. So, for agents $i$ who belongs to the $j$ th group, the investment decision $a_{i}(t)$ is no longer governed by $S_{i}(t)$, but instead is given by

$$
\begin{equation*}
a_{i}(t)=\operatorname{sgn}\left(u_{j}(t)\right), \tag{2.10}
\end{equation*}
$$

where $u_{j}$ is determined by the $j$ th group leader based on $\rho(t)$ and the previous gain history of the group.

A basic question is then how to design $u_{j}$ for good financial return. One important consideration factor is the
number of agents belonging to such groups. If this number is large, then the final market behavior may deviate significantly from the prediction given by $\rho$. A good control strategy may require estimating the number of agents belonging to such groups. It would be interesting to analyze the market dynamics of these types of models.

## 3. Air-Surface Cooperative Control Network

There is growing interest in the research of air-surface cooperative control systems which consist of a team of unmanned aerial vehicles (UAVs) interacting with a team of unmanned ground vehicles (UGVs). (For references see for example [15], [6], [17], [4], and [13].) There are many applications for these systems, including wide area surveillance, cooperative search, formation control, remote task coordination, and so on. It is stated in [23] that there are three cooperative scenarios in these systems, namely:

1) Aerial robots assist ground robots;
2) Ground robots assist aerial robots;
3) Ground and aerial robots cooperate to achieve a task.


Fig. 1: An air-surface network showing multiple UGVs jointly controlled by multiple UAVs. Similarly, multiple UGVs can control multiple UAVs. These scenarios demand complicated real-time communication networking structure.

In all these scenarios, communication among the robotic agents are complicated but indispensable. Assigning dedicated physical channels for each pair of agents is too costly if the number of agents is large and solutions via communication network become desirable. For shared communication channels, coordination of user access is a fundamental issue and there are many well researched solutions for networks with base stations, such as a cellular network. Although there is a natural two-layer structure for the network under consideration, there is no natural hierarchy among nodes in the same layer. For ad hoc networks like these, the traditional feedback-based, non-deterministic solutions such as CSMA quickly become unwieldy since every agent can serve as a base station and there is no natural way to uniquely nominate a base station set. Moreover, traditional media access control protocols are not intended for time critical applications. In spite of intense interests in developing real-time ethernet protocols, (see for example [40] and [12],) there are no obvious choice for communication protocols that can support mobile control networks such as the air-surface control networks described here. As real-time guarantee is hard to obtain, understanding the effect of information loss on control objectives becomes critical and much research has been reported on these issues (see for example [27], [28], [41],
[31], [9], and [11].) It is shown in [20] and [31] that systems with packet reception acknowledgments, referred to as TCPlike, have better performance than those without acknowledgments, referred to as UDP-like. Moreover, for TCP-like systems, classic separation principle holds, but not so for UDP-like systems. Unfortunately, providing acknowledgments in an ad hoc network is costly as these acknowledgments are additional data traffic.

The approach taken in [10], which relies on protocol sequences, offers a promising new direction of investigation. Further details on this novel approach is provided in Section 5.

## 3 KEY FEATURES OF NOI

These examples serve to illustrate the breadth of problems that can be modeled by an NOI and demonstrate that there can be great variations in the underlying dynamic models for these networks. Therefore, instead of making an attempt to define here an all inclusive formal structure of an NOI, we highlight some key features of these networks:

1) Control objectives may vary from time to time and are dependent on parameters, referred to as choices, that may not be universally known to all agents.
2) Locality and network connectivity can evolve dynamically.
3) Nonclassical information structure is the norm rather than exception. The separation principle of control and estimation cannot be guaranteed in general.
4) Contention in communication and computing resources may lead to varying delays in control actions.
5) No central coordinating authority, all control algorithms are distributed in nature and may only depend on information from local neighbors.

Not all these features are unique to NOI. For example, many distributed control systems are premised on timevarying locality and connectivity. Two of the more discriminating features are the time-varying, choice-based nature of control objectives and the influence of communication resource sharing on the effect of controls. Both features call for solutions that require communication through the network. In the following sections, we introduce concepts that can be invoked to address some related issues.

## 4 CONTROL COMMUNICATION COMPLEXITY FOR SYSTEM WITH CHOICES

In traditional distributed control problems, the control objective is typically assumed to be known to all agents. For example, for the consensus problem the objective is to derive distributed algorithms to enable agents reach a pre-defined consensus, such as, identical velocity for a fleet of UAVs. Generalized versions, such as the models covered in [38] and [25], where the goal is to ensure agents converge to different consensus clusters based on their choices pertain to the realm of NOI.

The papers [34], [35], [36], and [2] report results from early investigations on systems allowing choices. Some of these ideas and techniques can be extended to NOI.

Consider a controlled dynamical system of the form:

$$
\left\{\begin{array}{l}
\frac{d}{d t} \mathbf{x}(t)=\mathbf{f}\left(\mathbf{x}(t), \mathbf{u}_{A}\left(t, C_{A}, \tilde{\mathbf{y}}_{A}(t)\right), \mathbf{u}_{B}\left(t, C_{B}, \tilde{\mathbf{y}}_{B}(t)\right)\right) \\
\mathbf{x}(0)=\mathbf{x}_{0} \in \mathbb{R}^{N} \\
\mathbf{y}_{A}(t)=\mathbf{h}_{A}(\mathbf{x}(t)) \in \mathbb{R}^{L_{A}} \\
\tilde{\mathbf{y}}_{A}(t)=\left\{\left(s, \mathbf{y}_{A}(s)\right), 0 \leq s \leq t\right\} \\
\mathbf{y}_{B}(t)=\mathbf{h}_{B}(\mathbf{x}(t)) \in \mathbb{R}^{L_{B}} \\
\tilde{\mathbf{y}}_{B}(t)=\left\{\left(s, \mathbf{y}_{B}(s)\right), 0 \leq s \leq t\right\} \\
\mathbf{z}\left(t, c_{A}, c_{B}\right)=\mathbf{c}(\mathbf{x}(t)) \in \mathbb{R}^{M} \tag{4.11}
\end{array}\right.
$$

where f is a smooth vector field describing the inherent dynamics of the network. For simplicity, we consider the case with only two active agents, Alice and Bob. Alice controls the network via $\mathbf{u}_{A}$, which is dependent on Alice's target choice, $c_{A}$, and past history of the observation received by Alice up to $t, \tilde{\mathbf{y}}_{A}(t)$. Similarly, the control of Bob, $\mathbf{u}_{B}$, is based on $c_{B}$ and $\tilde{\mathbf{y}}_{B}(t)$. The target choice variables, $c_{A}$ and $c_{B}$, take values in the respective sets of choices, $\mathcal{A}=\{1, \ldots, m\}$ and $\mathcal{B}=\{1, \ldots n\}$.

The control objective is to steer the network to a chosen state at a finite terminal time, $T_{f}$, which is selected based on $c_{A}$ and $c_{B}$. At time $t=0$, Alice and Bob each selects a value for his or hers choice variable. The probability of selection is specified by distributions $\rho_{A}$ and $\rho_{B}$ respectively; both distributions are known to all agents. However, at time $t=$ 0 , Bob does not know what choice Alice has selected and vice versa. Let $\mathbf{H}(i, j)$ represent the target state when Alice selects $i$ and Bob selects $j$. Following [34], we define the target matrix to be the $m$ by $n$ matrix, $\mathbf{H}$ defined by:

$$
\begin{equation*}
\mathbf{H}_{i, j}=\mathbf{H}(i, j) . \tag{4.12}
\end{equation*}
$$

The control objective can be rephrased as to derive control law sets, $\mathcal{U}_{A}=\left\{\mathbf{u}_{A}(\cdot, 1), \ldots, \mathbf{u}_{A}(\cdot, m)\right\}$ and $\mathcal{U}_{B}=$ $\left\{\mathbf{u}_{B}(\cdot, 1), \ldots, \mathbf{u}_{B}(\cdot, n)\right\}$ with $\mathbf{u}_{A}$ dependent on $c_{A}$ and $\tilde{\mathbf{y}}_{A}(t)$ only and $\mathbf{u}_{B}$ dependent on $c_{B}$ and $\tilde{\mathbf{y}}_{B}(t)$ only, so that

$$
\begin{equation*}
\mathbf{z}\left(T_{f}, i, j\right)=\mathbf{H}_{i, j} \tag{4.13}
\end{equation*}
$$

As in the formulation of the standard terminal state control problem (single target), one may add control cost to the consideration. For example, one may be interested to find the optimal controls that minimize:

$$
\begin{gather*}
I\left(\mathcal{U}_{A}, \mathcal{U}_{B}\right)= \\
\sum_{i=1}^{m} \rho_{A}(i) \int_{0}^{T_{f}} \mathbf{u}_{A}^{T} \mathbf{u}_{A}(t, i) d t  \tag{4.14}\\
+\sum_{j=1}^{n} \rho_{B}(j) \int_{0}^{T_{f}} \mathbf{u}_{j}^{T} \mathbf{u}_{j}(t, j) d t
\end{gather*}
$$

subject to condition (4.13) is satisfied.
Although the control laws defined here are supposed to be dependent on the whole observation history, in an NOI, the observers and controllers are not necessarily co-located but connected via a digital communication network. Hence, there is a discrete time version of (4.11) so that observations and controls are dependent on coded messages. Details can be found in [34] and is not repeated here.

Noted that if the agent choices are known to both agents, the network terminal control problem can be decomposed into $m n$ standard terminal control problems. The mean control cost of these standard terminal control problems provides a lower bound for $I\left(\mathcal{U}_{A}, \mathcal{U}_{B}\right)$. This fact also offers
a solution approach whereby the control is divided into two phases. The objective of the first phase is to enable the agents to signal their selected choices to each other so that by the end of the phase, the problem is reduced to a standard terminal control problem; the solution is then applied in the second phase. For a given choice set $(i, j)$, the control cost of this type of solution is a sum of the cost in the signaling phase plus the cost of a standard terminal control problem, (with correspondingly adjusted initial time and initial state.)

At the other end are solutions which do not make use of any observations and hence there is no possibility of determining the other agent's choice. At first sight, this seems to be asking for an infeasible solution. Indeed, not every network terminal control problem allows for such type of solutions. But solutions can exist, although the control costs tend to be higher. In a sense, one can trade communication cost with control cost. Between these two extremes, that is communication-free on one hand and full signaling of the agents' choices on the other, are the hybrid solutions which allow agents to gradually learn more about each other's choices. For these solutions, the controls also carry information in addition to satisfying trajectory goals. (See [29] and [30] for ideas on communication through control.)

We note in passing that for the discrete time models, a natural question is how many information bits need to be exchanged before the target can be realized. By extending Yao's concept of communication complexity ([39] and [22]), a related concept of control communication complexity was introduced in [34] and further developed in [35].

Communication-free solutions are of interest as they offer solutions without requiring agent-to-agent communication. Whether such a solution exists is hinged on the inherent system dynamics and well as on the structure of the target matrix.

In [36], this question was resolved for the case where the output $z(t)$ is a scalar function that is bilinear in the controls $\mathbf{u}_{A}$ and $\mathbf{u}_{B}$. Systems satisfying this bilinear condition include the Brockett-Heisenberg system and linear systems of the form [36]:

$$
\left\{\begin{array}{l}
\frac{d \mathbf{x}(t)}{d t}=\mathbf{A} \mathbf{x}(t)+u(t) v(t) \mathbf{b}, \mathbf{x}(0)=\mathbf{0} \in \mathbb{R}^{N}  \tag{4.15}\\
y_{A}(t)=y_{B}(t)=z(t)=\mathbf{c}^{T} \mathbf{x}(t) \in \mathbb{R}
\end{array}\right.
$$

We note that in the smart grid model, if the phases are relatively constant and the voltages $V_{i}$ 's are regarded as controls, there is a natural connection between equation (2.1) and (4.15).

The bilinear assumption implies that the target realization problem is to find a factorization of $\mathbf{H}$ as a product of three matrices:

$$
\begin{equation*}
\mathbf{H}=\mathbf{U F V}{ }^{T}, \tag{4.16}
\end{equation*}
$$

where $\mathbf{U}$ and $\mathbf{V}$ are matrix representations of the controls in relation to an orthogonal base in the control space, while the matrix $\mathbf{F}$ is a description of the system input-output mapping. A basic result states that whether a communicationfree solution exists for the bilinear model hinges on the condition whether the rank of $\mathbf{F}$ is larger than or equal to that of $\mathbf{H}$. If it is, then communication-free solutions exist, otherwise they do not. Moreover, under suitable technical conditions, it is possible to derive the optimal control cost. When
compared with the mean control cost of the standard terminal control problems, these results allow us an inherent way to evaluate the value of the communication bits.

For the Brockett-Heisenberg system it can be shown that the rank of $\mathbf{F}$ is infinite, hence it can realize any finite target matrix without any communication. On the other hand, for a linear system, say of the form:

$$
\left\{\begin{array}{l}
\frac{d \mathbf{x}(t)}{d t}=\mathbf{A} \mathbf{x}(t)+u_{A}(t) \mathbf{b}_{A}+u_{B}(t) \mathbf{b}_{B}, \mathbf{x}_{0} \in \mathbb{R}^{N}  \tag{4.17}\\
y_{A}(t)=y_{B}(t)=z(t)=\mathbf{c}^{T} \mathbf{x}(t) \in \mathbb{R}
\end{array}\right.
$$

there exits a communication-free solution only if the target matrix satisfies the compatibility condition [34]:

$$
\begin{equation*}
H_{i k}-H_{j k}=H_{i l}-H_{j l} \tag{4.18}
\end{equation*}
$$

for $i, j \in\{1, \ldots, m\}$ and $k, l \in\{1, \ldots, n\}$.
For compatible target matrices, optimal communicationfree control laws were derived in [18]. These control laws are open-loop control. To handle state disturbances, it is desirable to use feedback control laws. One simple suggestion explored in [18] is to repetitively apply the open-loop control law using the latest state and time value as initial parameters while keeping the choice distribution fixed. In principle, such type of feedback laws can be shown to converge to the chosen targets. However, the control energy of these laws will diverge to infinity. An intuitive explanation is that the defined solution tries to perform the same task with shorter and shorter time duration and hence it requires an increasing amount of control cost.

A more sophisticated approach is investigated in [24] which allows the choice distributions be updated along with the state estimation. Unfortunately, the separation principle no longer holds for the combined system. Instead, suboptimal control laws were derived under the condition that Kitanidis estimators [21] are used. These control laws have finite control cost and perform well in numerical studies. We can regard these laws as examples of hybrid control laws alluded to earlier.
Finally a word on the non-compatible target matrix cases, for which there are no communication-free solutions. However, one can construct two-phase solutions as follows. In the first phase, which ends at time $t_{I}$, agents use the controls to indicate their choices. The output state at $t_{I}$ is coded to represent the true terminal state to be reached. Hence, in theory, each agent only needs to read the output information once to complete the solution. The value of $t_{I}$ needs to be fine-tuned so that the sum of the control costs at the two phases is minimized.

Some of these results can be extended to multi-agent models to offer new methodologies to study NOI. A difficulty to overcome is the "curse of dimensionality" as the target matrix will be replaced by a high dimensional tensor. However, if the objective is to understand global network behavior there are techniques to reduce the problem complexity.

## 5 REAL-TIME MEDIA ACCESS CONTROL

In [3], the concept of communication sequence was introduced to define the communication access order of sensors and actuators. A communication sequence is a periodic
binary sequence that can be used to assign communication tasks in a network control system. In communication theory, there is a related concept of protocol sequence [26] which was introduced to define random access to a shared communication medium, such as a radio frequency band supporting multiple users.

Traditional approach to the medium access control (MAC) requires some type of conflict resolution if more than one agent transmits at a given transmission time slot. Feedback information such as channel status or protocol instructions need to be sent to the transmitting agents by a central authority. These feedback based solutions proved to be efficient for systems built on base stations, such as wireless LANs. However, for an ad hoc network where there are no clear designated base stations, these feedback based MAC algorithms are inefficient. Protocol sequences based solutions on the other hand avoid using any feedback information at the MAC protocol layer. (High level acknowledgments may still be needed and can be implemented at a higher protocol layer with lower frequency.) The basic idea is to assign to each transmitting agent a unique periodic binary sequence. A physical channel is divided into time slots and at the beginning of every time slot a bit from the assigned sequence is read - a " 1 " represents permission to transmit and a " 0 " represents instruction to remain silent. If the sequences are designed with low cross correlation, it is possible to guarantee that no matter what time offsets the agents experience, any one of them can transmit at least one data slot without conflict from the other agents within one common period. This property is referred to as User Irrepressibility, or UI for short.

One drawback of this elegant idea is that the common period of protocol sequences tends to be very long. Using the original constructions in [26], supporting $M$ active agents with UI property requires sequences with a common period of at least $2^{M}$. Recent works have been focused on finding shorter UI sequences, for example [33], [32], and [37]. These works establish that the shortest known UI sequence sets have common periods in the order of $M^{2}$, where $M$ represents the number of active users.

Protocol sequence based MAC is particularly suitable for control network applications since it is possible to bound the maximum delay caused by contention from peer agents. TCP-like protocols, which support acknowledgments, are more powerful than UDP-like protocols, which do not have such acknowledgments, (see [31] .) However, in an ad hoc network, there is no simple way to guarantee acknowledgments are well-received as they are also data transmissions and require acknowledgments themselves. A feedback-free MAC can avoid this ad infinitum recursion.

To illustrate the use of protocol sequence in network control consider a simple model in which a controller aims to stabilize a remote system by sending control information over a shared wireless channel.

The basic dynamics of the system is of the form:

$$
\left\{\begin{array}{l}
x_{n+1}=c x_{n}+w_{n}+\alpha_{n} u_{n}, x_{0} \in \mathbb{R}  \tag{5.19}\\
y_{n+1}=x_{n+1}
\end{array}\right.
$$

Here $w_{n}$ 's represent state noises which are independent identically distributed random variables with mean 0 and fi-
nite variance. The variable $\alpha_{n}$ has value 1 or 0 indicating whether the control signal at time $t$ is received and applied or not. For simplicity, we assume that there is no observation noise and that the observation measurements are always available to the controller.

If the link between the controller and the actuator is subject to contention losses that can be modeled by independent Bernoulli random variables, $\left\{\alpha_{n}\right\}$, with probability that the transmission is successful given by $1-p$, then equation (5.19) is a simple variant of the model considered in [20]. If control cost is ignored, it is obvious that the optimal control is simply:

$$
\begin{equation*}
u_{n}=-c y_{n} . \tag{5.20}
\end{equation*}
$$

It follows from the arguments in [20] that the sequence $\left\{E\left[x_{n}^{2}\right]\right\}$ is bounded if and only if

$$
\begin{equation*}
c<\frac{1}{\sqrt{p}} \tag{5.21}
\end{equation*}
$$

On the other hand, if a MAC protocol based on a UI sequence set is used, the sequence $\left\{\alpha_{n}\right\}$ in equation (5.19) is no longer random but is deterministic once the time offsets among the agents are fixed. Let the period of the protocol sequences be $L$. By the UI property, there exists at least one contention-free transmission, $n$, between 0 and $L-1$. Under the assumption that peer contention is the only packet loss factor, this implies

$$
\begin{equation*}
\alpha_{n}=1 . \tag{5.22}
\end{equation*}
$$

Since the time offsets among agents remain unchanged, it follows that for $j=0,1, \ldots$,

$$
\begin{equation*}
\alpha_{n+j L}=1 \tag{5.23}
\end{equation*}
$$

Moreover,

$$
\begin{equation*}
x_{n+j L+1}=w_{n+j L} . \tag{5.24}
\end{equation*}
$$

Hence the sequence $\left\{E\left[x_{n}^{2}\right]\right\}$ is bounded for all values of $c$.
This simple example serves to illustrate the attractiveness of using protocol sequences in control schemes. More realistic scenarios incorporating control cost and sensing data loss are being investigated.

## 6 CONCLUSION

Networks of open interaction are rooted and evolved from information-based control and distributed control systems. They also take cues from the Internet and the Internet of Things. Remote enabling of actions and reactions on an open access platform will engender novel applications and services we have not yet dreamed of. However, this ambitious goal is still a distant target. Paths leading to it will be long and arduous, yet hopefully, rewarding.

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