CEO: Consistency of Encoding and Overhearing in Network Coding-Aware Routing

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***Abstract*—Network coding-aware routing attempts to find coding opportunities between an incoming flow and existing flows in wireless ad hoc networks. However, we find that a problem of most proposed coding-aware routing schemes is that they may misidentify the coding opportunities. Specifically, the opportunity misidentification may appear when a packet is about to be re-encoded. We call this the re-encoding problem. To solve this problem, we propose a principle called consistency of encoding and overhearing (CEO). By adhering to CEO, the encoding node is able to avoid misidentifying coding opportunities, thus ensuring the successful decoding of all encoded packets. In this letter, we first show the importance of CEO by analysis in a simple topology. Then, we introduce a distributed way to apply CEO to a coding-aware routing scheme, and show the improved network performance with CEO by simulation in different scenarios.**

***Index Terms*—Network coding-aware routing, wireless ad hoc networks, misidentified coding opportunity, re-encoding problem.**

I. Introduction

NETWORK coding [1], as a promising technique, has been widely applied in wireless networks [7]. It allows the intermediate node to encode packets from different flows into a single mixed packet and broadcast it to different receivers. If each destination of those flows is able to retrieve the packet intended to it from the mixed/encoded packet, network performance will be improved by reducing the number of transmissions for the packet delivery. Then, to take advantage of network coding, network coding-aware routing [2]–[6] has been proposed in wireless ad hoc networks. The routing protocols in [3]–[6] find coding opportunities and identify encoding nodes between two flows along their routes to improve network performance. However, the main problem of these routing protocols is that they may misidentify coding opportunities between two flows. That is, if one packet from one flow is re-encoded, then, the newly mixed packet may be un-decodable. Here, re-encoding a packet means encoding a packet that was coded previously.

Fig. 1 shows an example in which the coding opportunity may be misidentified when the distributed coding-aware routing (DCAR) in [3] is used. In Fig. 1, node 3 is the encoding node for flow 1 (f1) and flow 2 (f2), and node 4 is the decoding node for f1 and encoding node for another flow pair, f1 and flow 3 (f3). In this example, a packet, P1, from f1 is encoded at node 3 with another packet, P2, from f2 to form packet, P1 $\oplus$ P2. Packet P1 $\oplus$ P2 is then decoded at node 4 to yield P1. After that, node 4 encodes P1 with P3 from f3 to form P1 $\oplus$ P3. Now, P1 is considered a re-encoded packet because it was previously encoded with P2 and now encoded with P3. However, this is a misidentified coding opportunity, since when the new formed packet, P1 $\oplus$ P3, reaches node 7, it will be un-decodable there. This is because node 7 overheard P1 $\oplus$ P2 from node 3, not P1. We refer to this coding opportunity misidentification as the re-encoding problem, since it only appears in packet re-encoding. This re-encoding problem will also occur in [4]–[6]. We omit the details here.

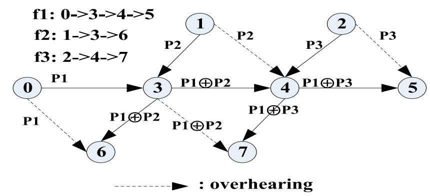

Fig. 1. Illustrative topology for the failure of network coding.

In fact, the re-encoding problem shown above will emerge more frequently as more flows in the network are intersected with each other. This is because when one flow (like f1 in Fig. 1) can be encoded with other two or more flows (like f2 and f3 in Fig. 1) at different intersection nodes, this re-encoding problem may appear. Due to space limit, we here do not show an example of this scenario in a figure.

One way to solve this re-encoding problem in [3]–[6] is to avoid misidentifying the coding opportunities in packet re-encoding. To achieve this, we propose a principle called consistency of encoding and overhearing (CEO) for the encoding node to adhere to. The root cause of the un-decodability in Fig. 1 is that there is an inconsistency between what was overheard at node 7, P1 $\oplus$ P2, and what is used to mix with P3, P1. Hence, for successful decoding, the encoding node needs to be aware that the two packets from two flows to be encoded must be the same two packets overheard by these two flows. This is the principle behind CEO.

Specifically, CEO restricts the behavior of some encoding node(s). For example, if we apply CEO to DCAR in Fig. 1, node 4 will deactivate the re-encoding of the packet from f1 and only encode f1's packets that are not encoded at node 3; whereas the encoding at node 3 will not be affected.

The contribution of this work is threefold. (1) We find the

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re-encoding problem widely exists in most proposed coding-aware routing schemes, and we propose the principle of CEO to solve it. (2) To show the importance of CEO to coding-aware routing [3]–[6] such as DCAR, we analyze the performance of DCAR and DCAR with CEO (DCAR-CEO) based on the topology in Fig. 1. (3) To further compare the performance of DCAR and DCAR-CEO in other various scenarios, we first propose a distributed way for the encoding node in DCAR to adhere to CEO, and then conduct simulations in different scenarios to show the advantage of DCAR-CEO.

II. Performance Analysis of DCAR and DCAR-CEO

We analyze the flow throughput of DCAR and DCAR-CEO based on the topology in Fig. 1 at different flow rates, $r_1,\ r_2,\ r_3$, of f1, f2, and f3, respectively. We employ the resource allocation or scheduling for the analysis [2]. T_i^j and $T_i^{(j,\ k)}$ are defined as the time fraction assigned to the unicast transmission from node *i* to node *j* and the broadcast transmission from node *i* to nodes *j* and *k*, respectively. This broadcast refers to the encoded packet transmission at node 3 or node 4 in Fig. 1. For interference-free scheduling, we emphasize that only the transmission from node 0 to node 3 and the transmission from node 2 to node 4 can be scheduled in parallel. In the analysis we assume each link capacity is 1, and without loss of generality, f3 is the last established flow.

DCAR: we only consider the rate combination $(r_1,\ r_2,\ r_3)$ within the rate region [8] of DCAR. Then, the throughput of f1 and f2 will be r_1 and r_2, respectively, whereas the throughput of f3 may be less than r_3 due to the decoding failure at node 7. Hence, we here focus on the throughput computation of f3.

Case 1: $r_1 \le r_2$. First, before f3 is set up, the resource allocation for f1 and f2 is as follows. For f1, $T_0^3 = T_4^5 = r_1$; for f2, $T_1^3 = r_2$ and $T_3^6 = r_2 - r_1$; for both f1 and f2, $T_3^{(4,\ 6)} = r_1$, because all packets from f1 are encoded with r_1/r_2 packets from f2 at node 3. Thus, due to the non-overlapping scheduling for all transmissions above for f1 and f2, we have the residual time fraction $T^{'} = 1 - 2r_1 - 2r_2$, which could be totally allocated to f3. Second, if f3 is set up with $r_3 \le r_1$, it will not occupy the residual time fraction, $T^{'}$, due to the parallel transmissions mentioned above and the coding opportunities between f1 and f3 at node 4. This is achieved by assigning $T_2^4 = r_3$ for f3, $T_4^5 = r_1 - r_3$ for f1, and $T_4^{(5,\ 7)} = r_3$ for both. Third, if f3 is set up with $r_3 > r_1$, extra time fraction from $T^{'}$ is needed for it, that is, we need additionally assign $T_2^4 = T_4^7 = r_3 - r_1$. Due to $T_2^4 + T_4^7 \le T^{'}$, we get the maximal achievable r_3 (r_3^{max}) $= r_1 + T^{'}/2 = (1 - 2r_2)/2$. Hence, by considering decoding failure at node 7, the throughput of f3, λ_3, is given by

$$\lambda_3 = \begin{cases} 0, & \text{if } 0 < r_3 \le r_1; \\ r_3 - r_1, & \text{if } r_1 < r_3 \le (1 - 2r_2)/2. \end{cases}$$

Under 1st condition $\lambda_3 = 0$ because all packets from f3 are encoded and thus un-decodable at node 7. Under 2nd condition $\lambda_3 = r_3 - r_1$ because a ratio of $(r_3 - r_1)/r_3$ packets from f3 are not encoded at node 4 and thus contribute to λ_3.

Case 2: $r_1 > r_2$. First, before f3 is set up, the resource allocation for f1 and f2 is as follows. For f1, $T_0^3 = T_4^5 = r_1$, and $T_3^4 = r_1 - r_2$; for f2, $T_1^3 = r_2$; for both f1 and f2, $T_3^{(4,\ 6)} = r_2$. Thus, we get $T^{'} = 1 - 3r_1 - r_2$. Second, similar to Case 1, by considering $T^{'}$ for f3, we get $r_3^{max} = r_1 + T^{'}/2 = (1 - r_1 - r_2)/2$. However, λ_3 in this case is different from that in Case 1, because a ratio of $(r_1 - r_2)/r_1$ packets from f1, i.e., those packets not encoded at node 3, can be encoded at node 4 with packets from f3. We consider the optimal case that this encodable portion of packets from f1 are encoded prior to the rest portion at node 4. Thus, we get $\lambda_3 = r_3$ when $0 < r_3 \le r_1 - r_2$. When $r_1 - r_2 < r_3 \le r_1$, we have $\lambda_3 = r_1 - r_2$, because only the opportunities with the encodable portion of f1 contribute to λ_3. When $r_1 < r_3 \le r_3^{max}$, we get $\lambda_3 = r_3 - r_2$ because a ratio of r_2/r_3 packets from f3 are encoded at node 4 and un-decodable at node 7. Thus, λ_3 is given as

$$\lambda_3 = \begin{cases} r_3, & \text{if } 0 < r_3 \le r_1 - r_2; \\ r_1 - r_2, & \text{if } r_1 - r_2 < r_3 \le r_1; \\ r_3 - r_2, & \text{if } r_1 < r_3 \le (1 - r_1 - r_2)/2. \end{cases}$$

Clearly, by characterizing the rate region of $(r_1,\ r_2)\,|_{r_3=0}$, we can get each r_3^{max} at each $(r_1,\ r_2)$, and hence characterize λ_3. The region of $(r_1,\ r_2)\,|_{r_3=0}$ is given by $\Omega_1 \cup \Omega_2$:

$\Omega_1 = \{(r_1,\ r_2) : 2r_1 + 2r_2 \le 1, 0 \le r_1 \le r_2\}$;

$\Omega_2 = \{(r_1,\ r_2) : 3r_1 + r_2 \le 1, r_1 > r_2 \ge 0\}$.

DCAR-CEO: with the redefined behavior of node 4 by CEO in Section I, we analyze the flow throughput at each achievable $(r_1,\ r_2,\ r_3)$ in DCAR-CEO. Since there is no decoding failure here, the throughput of each flow is equal to its flow rate. We compute r_3^{max} at a given $(r_1,\ r_2)$ as follows.

Case 1: $r_1 \le r_2$. Before f3 is established, the resource allocation for f1 and f2 and the residual time fraction are the same as those in Case 1 of DCAR. With CEO, there is no coding opportunity at node 4 in this case. Since we can assign $T_2^4 \le r_1$ without occupying the residual time fraction, r_3^{max} can be obtained as follows. **(a)** If $T^{'} \le r_1$, then we have $r_3^{max} = T^{'} = 1 - 2r_1 - 2r_2$ by assigning $T_4^7 = T_2^4 = T^{'}$. Hence, we have $\lambda_3 = r_3$, for $0 < r_3 \le T^{'}$. **(b)** If $T^{'} > r_1$, then we have $r_3^{max} = (r_1 + T^{'})/2 = (1 - r_1 - 2r_2)/2$ by assigning $T_4^7 = T_2^4 = (r_1 + T^{'})/2$. Thus, we have $\lambda_3 = r_3$, for $0 < r_3 \le (r_1 + T^{'})/2$.

Case 2: $r_1 > r_2$. We also have the same resource allocation and the residual time fraction as those in Case 2 of DCAR before f3 is set up. With CEO, however, a ratio of $(r_1 - r_2)/r_1$ packets from f1 could be encoded at node 4 in this case. Since we can assign $T_2^4 \le r_1$ for free, we can have $r_3 \le r_1 - r_2$ without occupying $T^{'}$. Then, based on the resource allocation at $r_3 = r_1 - r_2$, we further assign $T^{'}$ to f3 to get r_3^{max} as follows. **(c)** If $T^{'} \le r_2$, we have $r_3^{max} = r_1 - r_2 + T^{'} = 1 - 2r_1 - 2r_2$ by additionally assigning $T_4^7 = T_2^4 = T^{'}$. Thus, we have $\lambda_3 = r_3$, for $0 < r_3 \le r_1 - r_2 + T^{'}$. **(d)** If $T^{'} > r_2$, then, we obtain $r_3^{max} = r_1 - r_2 + (T^{'} + r_2)/2 = (1 - r_1 - 2r_2)/2$ by additionally assigning $T_4^7 = T_2^4 = (T^{'} + r_2)/2$. Thus, we have $\lambda_3 = r_3$, for $0 < r_3 \le r_1 - r_2 + (T^{'} + r_2)/2$.

Similarly, we need to know the region of $(r_1,\ r_2)\,|_{r_3=0}$ to characterize r_3^{max} and λ_3 in DCAR-CEO. Obviously, DCAR

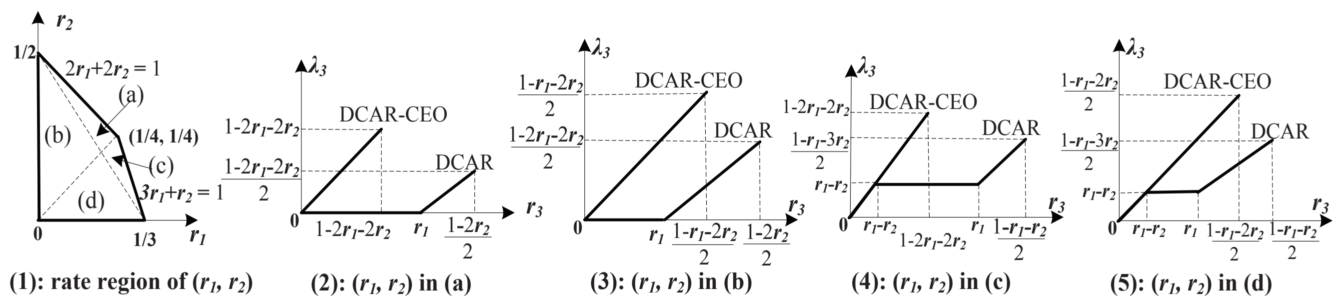

Fig. 2. The rate region of $(r_1,\ r_2)$ and the comparison between DCAR and DCAR-CEO.

and DCAR-CEO have the same region of $(r_1,\ r_2)|_{r_3=0}$, which is given by $\Omega_1 \cup \Omega_2$ and shown in Fig. 2(1).

Comparison: to plot λ_3 in DCAR and DCAR-CEO, we firstly divide the region of $(r_1,\ r_2)$ into four areas corresponding to the four situations in DCAR-CEO from **(a)** to **(d)**. Then, four figures, from Fig. 2(2) to Fig. 2(5), show λ_3 of the two schemes as the function of r_3 at any given $(r_1,\ r_2)$ in the four regions, respectively. From the four figures we have two observations. First, if $(r_1,\ r_2)$ is inside the rate region, DCAR-CEO basically outperforms DCAR, especially achieving a higher maximal λ_3 in all of the four figures. Second, interestingly, referring to Fig. 2(2) and Fig. 2(4), when one of the two boundaries ($2r_1+2r_2=1$ and $3r_1+r_2=1$) in Fig. 2(1) is reached, which means f1 and f2 have occupied all resource leaving $T^{'}=0$, both achieve the same maximal λ_3 (λ_3^{max}) such that $\lambda_3^{max}=0$ in Fig. 2(2) and $\lambda_3^{max}=r_1-r_2$ in Fig. 2(4). This indicates that the decoding failure at node 7 will not decrease λ_3^{max} in DCAR in this special situation.

In summary, this analysis shows CEO is important to coding-aware routing in that it ensures the packet decoding, and thus improves network performance in most situations.

III. A Distributed Way For Adherence of CEO

To adhere to CEO for the encoding node, it needs to know what packet from one flow was overheard by the other flow. To achieve this, we propose two new mechanisms and integrate them into DCAR to form DCAR-CEO.

A. Overheard Nodes Recoding

This mechanism allows the encoding node(s) of two flows, such as node 3 and node 4 in Fig. 1, to know which upstream nodes of one flow could be overheard by the downstream nodes of the other flow. This information can be directly obtained during the route discovery process in DCAR.

In DCAR, in order to identify a node as an encoding node between a new incoming flow and existing flows, each node must execute two tasks. First, each node on an existing flow must record the entire route of its flow. This could be achieved after the flow route is established [3]. Second, upon receiving the route request during route discovery for a new incoming flow, each node must include its one-hop neighbors in the route request before propagating it to the downstream nodes. With this complete route and one-hop neighbor information, each node could determine whether it is an encoding node during the route reply (for details, please refer to [3]).

In DCAR-CEO, the coding opportunity identification remains the same as in DCAR, but the identified encoding node is required to record the *upstream overheard nodes* of one flow that can be overheard by its downstream node(s) of another flow. In Fig. 1, node 3 and node 2, which are the overheard nodes on f1 and on f3, respectively, will be recorded by the intersection node 4 for the flow-pair, f1 and f3.

B. Packet Encoding/Decoding Recording

In DCAR-CEO, whenever packet encoding or decoding takes place at a node, that node is required to record that event in the packet header, which consists of two types of information. The first is the ID of that node. The second is the packet ID information, which indicates what packets are encoded after encoding or what packets are mixed if it is still an encoded packet after decoding. For example, in Fig. 1, node 1 will record two packet IDs, P1 and P2, together with node 1's ID in the encoded packet header. This record will be carried along the route by subsequent downstream nodes. Then, in the event of re-encoding or decoding for that packet, these two kinds of information for that new event will be appended in the packet header. With this record, any node on the flow path will know what packets are encoded together in the encoded packet transmitted by any of its upstream nodes. The overhead of this recording is limited because the size of this information is small and the number of encoding or decoding node is limited because of the small number of hops traversed by each packet.

C. Re-Encoding Decision at the Encoding Node

With the two mechanisms introduced above in DCAR-CEO, an encoding node will know what packets are transmitted by the overheard node(s) and what packets are overheard by the downstream node(s) of another flow. Hence, with the knowledge of what packets are overheard by the downstream node(s) of another flow, the encoding node can make accurate decisions on which packets to encode and which not according to CEO. For example, when node 4 in Fig. 1 is about to encode P1 and P3, it will refer to the encoding/decoding record in these two packet headers. From the record of P1, if node 4 finds out that the transmitted packet at node 3 is an encoded packet, P1 $\oplus$ P2, which is overheard at node 7, then it will not perform encoding because P1 $\oplus$ P2 is different from the packet, P1, to be encoded with P3 at it. Otherwise, node 4

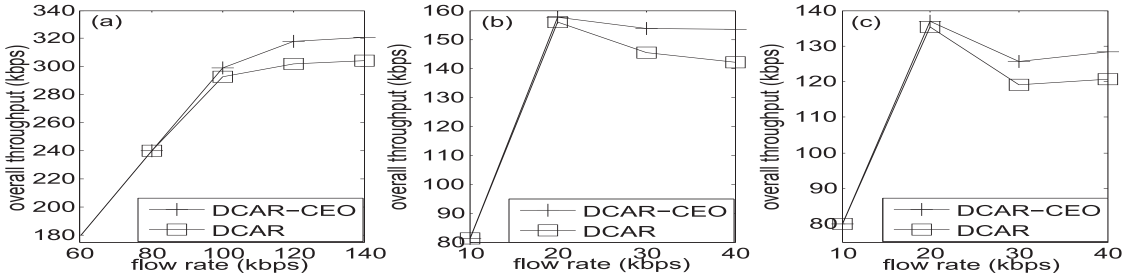

Fig. 3. Performance comparison between DCAR and DCAR-CEO in different scenarios.

will encode the two packets from f1 and f3. Thus, we see that with the two mechanisms and CEO, all coding opportunities can be correctly identified and thus be effectively utilized.

IV. Simulation Results

In this section, we evaluate the performance of DCAR and DCAR-CEO by simulation in NS-2. Instead of using scheduling, we adopt IEEE 802.11 standard in the MAC layer in our simulation. The radio transmission range is 100m and the data transmission rate is 1Mbps. User Datagram Protocol (UDP) is used for the traffic source. For simplicity, we assume all flows have the same flow rate.

Fig. 3(a) shows the overall throughput comparison between DCAR-CEO and DCAR in the topology of Fig. 1. First, as expected, DCAR-CEO improves the overall throughput by about 6% as compared to DCAR at high flow rate. Actually, we find that the throughput of f3 is improved significantly (by 30%) as compared to DCAR (not shown as a figure due to space limit). Obviously, the packet un-decodability at node 7 leads to the lower performance of DCAR. We find in the simulation that about 50% packets encoded at node 4 in DCAR cannot be decoded at node 7 at high flow rate. Second, the two schemes perform the same at low flow rate whereas DCAR-CEO outperforms DCAR in Fig. 2(2) and Fig. 2(3) at $r_1 = r_2 = r_3$ in the analysis. Since our simulation uses IEEE 802.11 which is a random-access MAC, not a scheduling based MAC, few coding opportunities will appear at node 3 between f1 and f2 at low flow rate. Thus, few packets from f1 will be re-encoded at node 4 with packets from f3 resulting in fewer decoding failures for f3 at node 7 in DCAR. Hence, DCAR-CEO performs the same as DCAR at low flow rate.

Next, we employ the 4×4 grid topology used in [3] for study. Eight flows will be randomly generated at different time. Fig. 3(b) shows the overall throughput of the 8 flows. It can be seen that DCAR-CEO outperforms DCAR. At high flow rate, 40kbps, the performance improvement of DCAR-CEO over DCAR is about 8%. This is because we find in this simulation that DCAR-CEO is able to decode all encoded or re-encoded packets whereas DCAR is not able to do this.

Lastly, we compare DCAR-CEO and DCAR in a 400m $\times$ 400m network with 30 nodes randomly displaced and eight flows randomly generated. Fig. 3(c) shows the performance comparison between them. Again, DCAR-CEO improves the overall throughput of DCAR by about 7% at high flow rate.

V. Conclusion

Our proposed CEO principle is fundamental to network coding-aware routing in that it guarantees successful decoding of all encoded or re-encoded packets as long as overhearing is reliable. We also propose two new mechanisms to facilitate the adherence of CEO for the encoding node. Specifically, with the two mechanisms, an encoding node is able to correctly identify each coding opportunity. By both analysis in a simple topology and simulations in various scenarios, we show that network performance can be improved by CEO.

Even though we study the performance of DCAR-CEO based on UDP traffic in this letter, we expect DCAR-CEO to show more significant performance improvement over DCAR for TCP traffic by avoiding lost TCP packets due to un-decodability. High packet loss rate is particularly detrimental to TCP performance because the TCP congestion control mechanism will interpret the packet loss as an indication of congestion (however in actuality, the majority of the loss is due to un-decodability, not congestion). The TCP setting and study will be explored in our future work.

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