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Stabilizing and Tracking Control of Multiple Pendulum-Cart Systems over a Shared Wireless Network

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Abstract: Interest in networked control systems has grown rapidly. In some remotely time critical control applications, multiple sensors and actuators are connected to controllers via a shared wireless network. Feedback and control information is transmitted over the network, and the time critical data traffic can tolerate only low bit error probability and bounded delays. This paper investigates control and communication of multiple dynamical systems in time critical networked control applications by accomplishing stabilizing and tracking control of multiple pendulum-cart systems over a shared wireless network. A testbed was developed consisting of three pendulum-carts remotely controlled by one embedded controller. The scheduling of the multiple plants accessing to the shared channel uses a time division multiplexing (TDM) scheme. PD control laws are deployed, meanwhile, Kalman filtering is considered to compensate for data packet loss in network transmission. Experimental results validate that the multiple pendulums can be stabilized while the carts accurately track the desired trajectories.

Key Words: Networked control, shared communication channel, multiple pendulum-carts, scheduling scheme, Kalman filter

1 Introduction

Networked control systems (NCSs) become increasingly important as many novel applications have been identified such as remotely controlling a network of mobile sensors [1-3]. These new applications can lead control engineering and network applications into a new era. In time critical network control applications, sensors, controllers and actuators are linked through communication networks. Feedback information and control commands, which we call time-critical data traffic, are transmitted over networks. The time-critical data traffic can tolerate only low bit error probability and constrained delays. Substantial effort has been put into developing communication protocols, control and coding schemes for NCSs. Because of data packet loss and communication delay inherent in NCSs, design and analysis of control laws and coding schemes have been studied for real-time applications [4-7]. On the other side, scheduling strategies have been presented to manage communication between controllers and plants [8-10].

Current research on NCSs over wireless channels has been mainly focused on the employment of a dedicated network. However, this arrangement can be costly or too complex if a large number of dynamical systems need to be controlled. Controlling dynamical systems through a shared network can provide significant improvement in efficiency and reduction of development cost. For example, controlling a swarm of pilotless planes or mobile robots by dedicated channels may require an unrealistic number of non-interfering radio channels. It is hence of interest to study NCSs with the communication between sensors, actuators and controllers being shared over the same wireless channel. A major challenge is determining how to schedule network resources of the shared network to accomplish different control tasks. Due to under-actuated, fast and unstable dynamics, the stabilizing and tracking control of a pendulum-cart system is a challenging task especially for wireless control. In this paper, we study issues of control and communication in time critical network control applications exemplified by accomplishing stabilizing and tracking control of multiple pendulum-cart systems over a shared wireless network. The stabilization of pendulum by a remote controller over a dedicated channel was examined in [11-12]. A. Suri et al. developed a real-time testbed with pendulum-cart system to study feedback over Bluetooth [11]. N. J. Ploplys et al. presented a timing scheme for wireless control loop and illustrated the stabilization of rotating based pendulums over 802.11b [12]. Compared to stabilizing one pendulum by one remote controller [11-12], we examine the stabilizing and tracking control of multiple pendulums by one controller over a shared wireless network. To compensate for the data packet loss inherent in NCSs, Kalman filtering is considered to improve the performance of the control laws.

The paper is organized as follows. In Section II, we describe the system configuration. Section III presents the control laws and the communication protocols. In Section IV, experimental results are presented and analyzed. Section V concludes the paper.

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2 System Description

To explore the remote control of dynamical systems over a shared wireless network for real-time networked control applications, a system consisting of multiple pendulum-cart systems is examined. The configuration of such a networked control system is illustrated in Fig. 1. Multiple pendulum-cart systems are stabilized by one remote controller while the sensor information and the computed control values are transmitted over a shared wireless network.

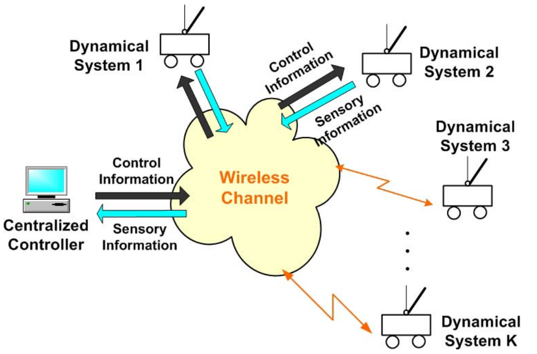


Fig. 1. Configuration of one centralized controller controlling multiple dynamical systems over a shared wireless channel.

The pendulum-cart platform is shown in Fig. 2. For each pendulum-cart system, the control force is executed via a DC geared motor to actuate the cart. The pendulum is mounted on the cart and reacts to the motion of the cart. The pendulum angle from the upright position and the cart position are measured via incremental encoders. The velocities of the pendulum and the cart are calculated numerically from the discrete measured encoder signals. The encoder signals of the *ith* pendulum cart are encoded and transmitted to the remote controller over a wireless network. Once the feedback information has been received, the controller computes the control values and transmits them to the actuator over the wireless channel.

Fig. 2. The pendulum-cart system with the system parameters: $M = 1.61\text{kg}$, $m = 0.09\text{kg}$, $L = 0.24m$, $I = 0.0017\text{kgm}^2$.

Without considering the friction, the pendulum-cart can be simplified as presented in Fig. 3. The parameters of the pendulum-cart are shown in Tab.1. The nonlinear dynamics of the pendulum-cart system can be described as [16]:

$$\begin{aligned}&(M+m)\ddot{x} + mL\cos\theta\cdot\ddot{\theta} - mL\sin\theta\cdot\dot{\theta}^2 = u,\\&\left(I+mL^2\right)\ddot{\theta} - mgL\sin\theta + mL\cos\theta\ddot{x} = 0.\end{aligned}\tag{1}$$

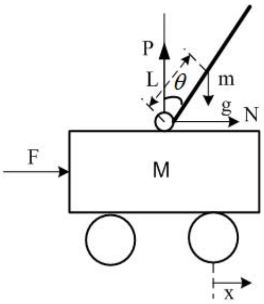


Fig. 3. Structure of a pendulum-cart system.

Table 1: Parameters of the pendulum-cart system

M	Mass of the cart
m	Mass of the pendulum
L	Half length of the pendulum
I	Inertial of the pendulum, $I = mL^2/3$
u	Applied force on the actuation
x	Cart position
θ	Pendulum angle from the upright position
g	Gravitational acceleration, $9.81 m/s^2$

Assume that $\theta << 1$，which implies that the pendulum is stabilized around the upright position. Then a good approximation can be obtained as $\cos\theta \approx 1$, $\sin\theta \approx \theta$, $\dot{\theta}^2 \approx 0$. The linearized dynamical equations are then given by:

$$\begin{cases}\left(I+mL^2\right)\ddot{\theta} - mgL\theta + mL\ddot{x} = 0,\\ \left(M+m\right)\ddot{x} + mL\ddot{\theta} = u.\end{cases}\tag{2}$$

Reorganizing (2), we have:

$$\begin{cases}\ddot{x} = -\dfrac{m^2L^2g}{I(M+m)+mML^2}\theta + \dfrac{I+mL^2}{I(M+m)+mML^2}u,\\ \ddot{\theta} = \dfrac{mgL(m+M)}{I(M+m)+mML^2}\theta - \dfrac{mL}{I(M+m)+mML^2}u.\end{cases}\tag{3}$$

From (3), the system transfer function is given:

$$\frac{\theta(s)}{U(s)} = \frac{K_p}{s^2/A_p^2 - 1},\tag{4}$$

where $K_p = \dfrac{1}{(M+m)g}$, $A_p^2 = \dfrac{mgL(M+m)}{(M+m)(I+mL^2)+m^2L^2}$.

The motor actuates the cart with the executed control force $u(t)$, which is related to the torque $\tau(t)$ and the electricity voltage $v(t)$ of the DC gear motor as [17]:

$$u(t) = \frac{n_r}{r}\tau(t) = \alpha v(t) - \beta\dot{x}(t),\tag{5}$$

where n_r is the gear ratio, r is the driver wheel radius, α and β are constants related to the DC motor parameters.

Using $x(t) = r\theta(t)/n_r$ and (5), the transfer function of the pendulum-cart system can be given by:

$$\frac{\theta(s)}{V(s)} = \frac{K_p\alpha}{s^2/A_p^2 + K_p r\beta/n_r\cdot s - 1}.\tag{6}$$

For the DC geared motors utilized in our platform, it has $\alpha \approx 1$ and $K_p r\beta/n_r \approx 0$. The system transfer function in (6) can thus be approximated as in (4), where the dynamical equation (5) of the DC motor does not explicitly appear.

Accordingly, in the paper, the dynamics in (3) is considered for the pendulum-cart system.

Let $\boldsymbol{x}=[\theta,\dot{\theta},x,\dot{x}]^T$. The state-space representation of the pendulum–cart system can be given by:

$$\dot{\boldsymbol{x}}=\begin{bmatrix}\dot{\theta}\\ \ddot{\theta}\\ \dot{x}\\ \ddot{x}\end{bmatrix}=\begin{bmatrix}0&1&0&0\\ a&0&0&0\\ 0&0&0&1\\ b&0&0&0\end{bmatrix}\begin{bmatrix}\theta\\ \dot{\theta}\\ x\\ \dot{x}\end{bmatrix}+\begin{bmatrix}0\\ c\\ 0\\ d\end{bmatrix}u=\boldsymbol{A}\boldsymbol{x}+\boldsymbol{B}u, \tag{7}$$
$$\boldsymbol{y}=\begin{bmatrix}\theta\\ x\end{bmatrix}=\begin{bmatrix}1&0&0&0\\ 0&0&1&0\end{bmatrix}\boldsymbol{x}+\begin{bmatrix}0\\ 0\end{bmatrix}u=\boldsymbol{C}\boldsymbol{x},$$

where $a=mgL\left(M+m\right)/\lambda$, $b=-m^2L^2g/\lambda$, $c=-mL/\lambda$, $d=\left(I+mL^2\right)/\lambda$, $\lambda=I\left(M+m\right)+MmL^2$.

With the sampling time T_s, the discrete-time system of the pendulum-cart system can be represented as:

$$\begin{aligned}\boldsymbol{x}[k+1]&=\boldsymbol{G}\boldsymbol{x}[k]+\boldsymbol{H}u[k],\\ \boldsymbol{y}[k]&=\boldsymbol{C}\boldsymbol{x}[k]+\boldsymbol{D}u[k],\end{aligned} \tag{8}$$

where $\boldsymbol{x}=[\theta,\Delta\theta,x,\Delta x]^T$, $\boldsymbol{G}=e^{AT_s}$, $\boldsymbol{H}=\int_0^{T_s}e^{A(T_s-\tau)}\boldsymbol{B}d\tau$.

3 Communication and Control

In this section, stabilizing and tracking control of three pendulum-cart systems by one remote controller over a shared wireless network is examined. A PD control law is considered. Kalman filtering is deployed to improve the performance of control laws. A scheduling scheme of accessing to the channel for the remote controller and the multiple plants is presented.

Let $\boldsymbol{x}=[\theta,\Delta\theta,x,\Delta x]^T$ represents the system states and $\boldsymbol{x}_\text{d}=\left[\theta_d,\vartriangle\theta_d,x_d,\vartriangle x_d\right]^T$ denote the desired states, where $\vartriangle\theta[k]/\vartriangle\text{x}[k]$ is the velocity of the pendulum angle and cart position, respectively. The state error vector $\boldsymbol{e}[k]$ is:

$$\boldsymbol{e}[k]=\boldsymbol{x}_d[k]\text{-}\boldsymbol{x}[k].$$

A PD control strategy is considered for stabilizing and tracking control of the pendulum-cart system:

$$u[k]=-(\boldsymbol{k}_p\boldsymbol{e}[k]+\boldsymbol{k}_d\vartriangle\boldsymbol{e}[k]), \tag{9}$$

where $\vartriangle\boldsymbol{e}[k]=(\boldsymbol{e}[k]-\boldsymbol{e}[k-1])/T_s$, and $\boldsymbol{k}_p$ and $\boldsymbol{k}_d$ denote the proportional and derivative parameters, respectively.

The velocities $\Delta\theta$ and Δx of pendulum angle and cart position are computed numerically using a difference scheme. The sensor packets may be lost when sent to the controller over a wireless channel. To mitigate the adverse effects of packet loss and numerically computed velocities, a moving average filter is used to smooth the velocities:

$$\begin{aligned}&\vartriangle\hat{\theta}[\text{k}]=0.6\vartriangle\theta[\text{k}]+0.3\vartriangle\theta[\text{k-1}]+0.1\vartriangle\theta[\text{k-2}],\\ &\vartriangle\hat{\text{x}}[\text{k}]=0.6\vartriangle\text{x}[\text{k}]+0.3\vartriangle\text{x}[\text{k-1}]+0.1\vartriangle\text{x}[\text{k-2}].\end{aligned} \tag{10}$$

The computed control value $u[k]$ suffers random and abrupt variations due to observation noise as well as data packet loss inherent in network transmission. To relieve these unfavorable effects, Kalman filtering is considered to further improve the performance of the PD control laws.

The Kalman filtering algorithm is formulated as follows:

$$\begin{aligned}&\hat{u}^-[k]=\hat{u}[k-1],\\ &P_k^-=P_{k-1}+Q,\\ &K_k=P_k^-(P_k^-+R)^{-1},\\ &\hat{u}[k]=\hat{u}^-[k]+K_k(u[k]-\hat{u}^-[k]),\\ &P_k=(I-K_k)P_k^-,\end{aligned} \tag{11}$$

where Q/R is the process/observation noise covariance, respectively, and $\hat{u}[k]$ denotes the filtered control values.

The communication protocol considered in the paper does not provide acknowledgement (ACK) of successful delivery of control packets to the controller. It is shown in [13-15] that the classic separation principle does not hold without or even with partial ACK of control packets. Estimation and control are coupled, and design of optimal or suboptimal estimator and controller presents a highly complex problem [14-15]. For simplicity, Kalman filtering in (11) is deployed in the paper to compensate for the data packet loss.

The diagram of the closed-loop feedback system for the *i*th pendulum-cart system with the PD controller and the Kalman filtering scheme is shown in Fig. 4.

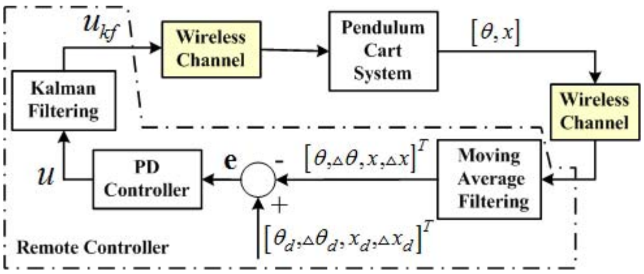


Fig. 4. The diagram of the closed-loop feedback system.

In one sampling interval, the channel source is allocated to the three pendulum-carts using a time division multiplexing (TDM) scheme. Fig. 5 depicts the timing diagram in one sampling interval. The remote controller sends *Ready* signal to the carts based on the time trigger mode. The i*th* cart begins to sample the sensor data once it receives the *Ready* signal, and the sampling of the i*th* cart is based on the event trigger mode. The feedback information including pulses number of the encoder increments as well as the direction of the cart and the pendulum is then sent to the remote controller via a wireless channel. The time delay for the sampling to get the sensor data is assumed to be small relative to the transmission time. Once the feedback information has been received, the controller computes the control commands and transmits them to the actuator over the wireless channel. The process for the computing and transmitting the control values is based on the event trigger mode. The computational time can be neglected compared with the transmission delay.

The communication between the controller and the plant consists of two downlinks (controller to cart) and one uplink (cart to controller). For the platform under consideration, such a complete communication takes about 12 milliseconds (ms) with the transmitting data rate 57.6 kbps. Timing inaccuracies of micro control units may accumulate timing errors and lead to channel access collision. A guard time is added for each plant to prevent the channel access

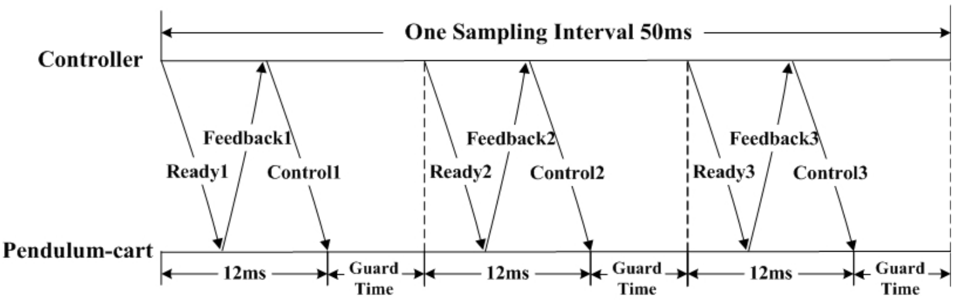


Fig. 5. The timing diagram in one sampling period.

collision. Considering the sampling interval 50ms, the time accessing to the channel for each cart is around $50/3 \approx 16.6667\text{ms}$. After the *Ready* signal has been sent by the controller, i.e., 16.66667ms later, the process for the i*th* pendulum-cart mentioned above is repeated for the other two pendulum- cart systems. With this channel access scheduling, each cart can ensure to send feedback information and to receive control commands to/from the remote controller within one sampling interval without resorting to high precision clock synchronization.

In order to identify different pendulum-carts and decrease the transmission error, the transmission data are encoded before sent over the wireless channel. The data packages structures for the downlink and uplink communications between the controller and the plants are shown in Fig. 6.

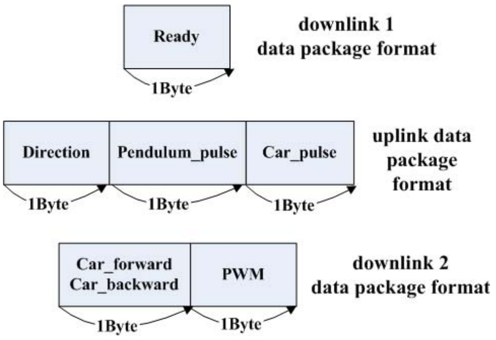


Fig. 6. Data packages structures for the downlink and uplink communications.

4 Experimental Results

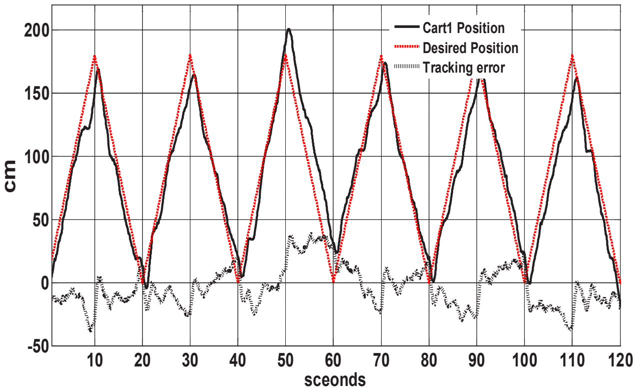
The control and communication of multiple plants over a shared wireless network is investigated by accomplishing stabilizing and tracking control of multiple pendulum-carts by a remote controller. Experiments were performed to validate the presented control laws and scheduling schemes. The testbed consists of one embedded controller using a Freescale control unit and three pendulum-carts as shown in Fig. 2. The sampling interval should be lager than 15.4 Hz (less than 65 ms) to stabilize the pendulum. The sampling interval is set as 50ms. Wireless radio frequency (RF) modules which support RS232 communication are used for wireless transmission. Under the transmission data rate 57.6 kbps of the wireless RF modules, the round-trip time is about 12ms for each plant. The pendulum angle is measured by an incremental encoder with 1000 pulses/rev. The cart position is measured by an incremental encoder with 500 pulses/rev. It is mounted on a follower wheel.

By setting the desired system states $\boldsymbol{x}_\text{d}$, the pendulum can remain stabilized ($\theta_d = 0$) while the cart position tracks the desired trajectories. The PD control parameters are set as $\boldsymbol{k}_p = [160, 20, 45, 48]$ and $\boldsymbol{k}_d = 0.001\boldsymbol{k}_p$. The parameters of the Kalman filter are $Q = 0.001$ and $R = 0.85$. Fig. 7 depicts the cart position x and the pendulum angle θ. It can be seen that the pendulum is stabilized while the cart position accurately tracks the desired sawtooth wave. Fig. 9 and Fig. 11 display the experimental results of the pendulum- carts 2and 3, respectively. Similarly, for the pendulum-carts 2 and 3, it can be shown that the pendulums can be stabilized while the carts show good tacking ability of the desired sinusoidal and triangular waves, respectively. The stabilizing and tracking performance is related to the resolution of the incremental encoders, and can be improved using encoders with higher resolution.

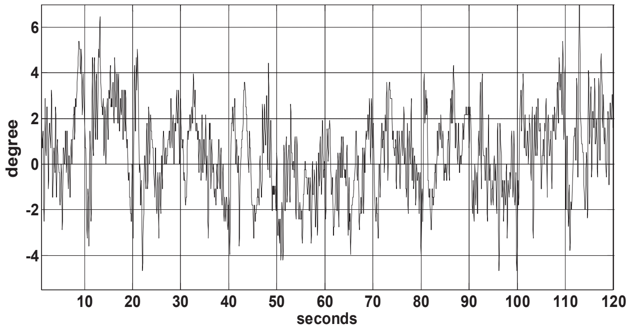
Fig. 8(a), Fig. 10(a) and Fig. 12(a) depict the computed control values using the PD control law, while Fig. 8(b), Fig. 10(b) and Fig. 12(b) illustrate the control values applied to the DC motors after the Kalman filtering. Due to data packet drops in network transmission as well as observation noise, the control values experience random and abrupt variations as shown in Fig. 8(a), Fig. 10(a) and Fig. 12(a). The carts move with noticeable jitter actuated by these control values. On the other hand, as shown in Fig. 8(b), Fig. 10(b) and Fig. 12(b), the filtered control values are much smoother. In the experiments, it can be observed that the carts move with even motion. The Kalman filter helps to compensate for sensor data packet loss and smooths the control values. The control values may also be lost when sent to the actuators over a wireless channel. At the *k*th time, the control $u[k-1]$ is applied instead when $u[k]$ is missed. Since the filtered control values are much smoother, such a substitution becomes realistic.

5 Conclusions

In some real-time networked control applications, feed-back and control information for multiple plants are sent over a shared wireless network. The issues of control and communication of multiple plants over a shared wireless network were explored via studying stabilizing and tracking control of multiple pendulum-carts by one remote controller. Experimental results validate the presented control law, filtering algorithms and the scheduling scheme. It is shown that the multiple carts can accurately track the desired trajectories with the pendulums being stabilized. Optimal control under packet loss statistics will be studied. Efficient quantization and coding schemes as well as communication protocols for NCSs will be explored in the future work.

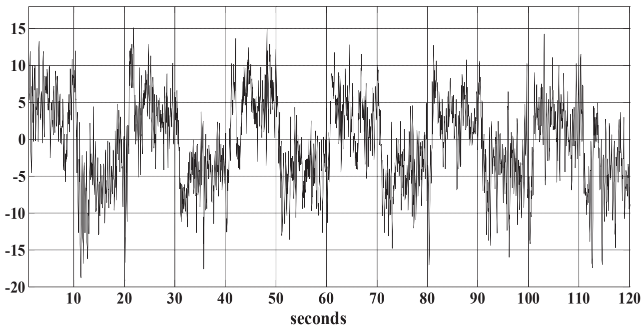


(a) Cart 1 position.

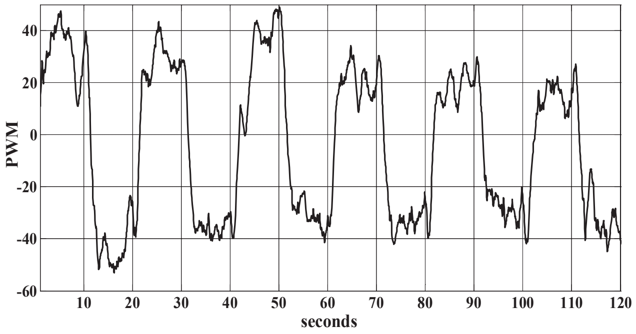


(b) The pendulum 1 angle θ .

Fig. 7. Experimental results for the cart 1, where the desired trajectory is a sawtooth wave. (a). The actual trajectory, desired trajectory and tracking error of the cart. (b) The pendulum angle θ .

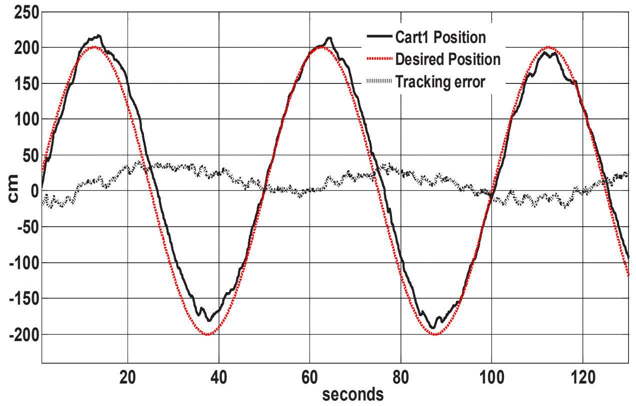


(a) Control values of the cart 1 before Kalman filtering.

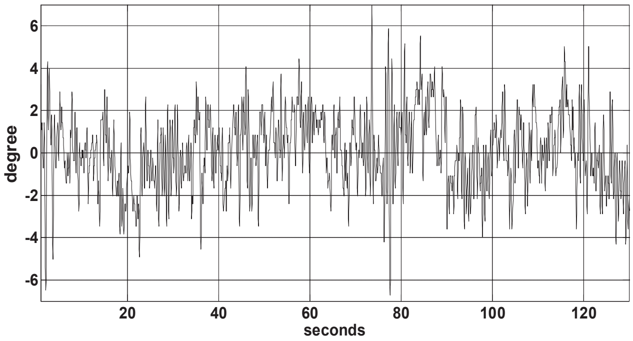


(b) Control values of the cart 1 after Kalman filtering.

Fig. 8. (a) Control values of the cart 1 before Kalman filtering. (b) Control values of the cart 1 after Kalman filtering.

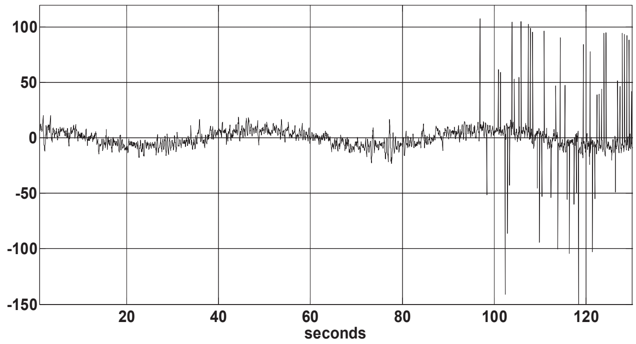


(a) Cart 2 position.

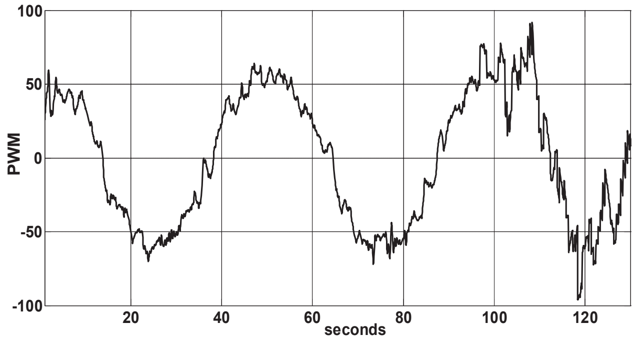


(b) The pendulum 2 angle θ .

Fig. 9. Experimental results for the cart 2, where the desired trajectory is a sinusoidal wave. (a) The actual trajectory, desired trajectory and tracking error of the cart. (b) The pendulum angle θ .

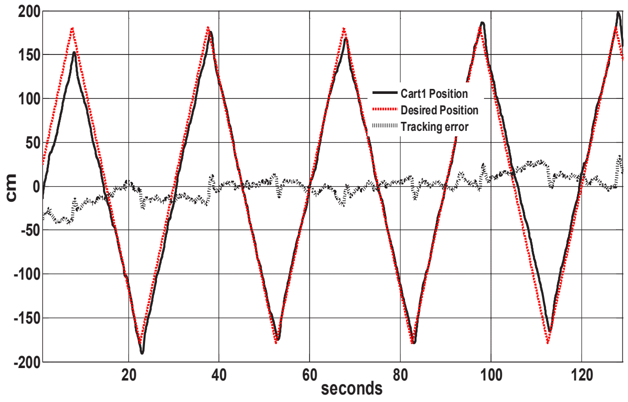


(a) Control values of the cart 2 before Kalman filtering.

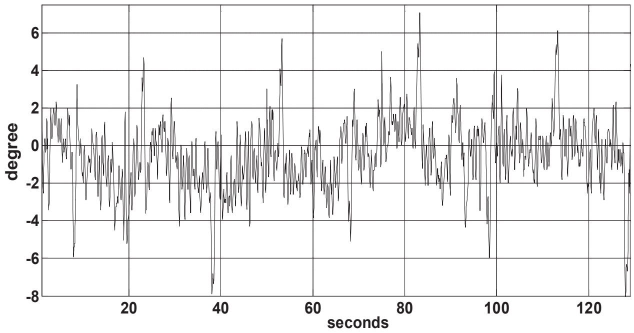


(b) Control values of the cart 2 after Kalman filtering.

Fig. 10. (a) Control values of the cart 2 before Kalman filtering. (b) Control values of the cart 2 after Kalman filtering.

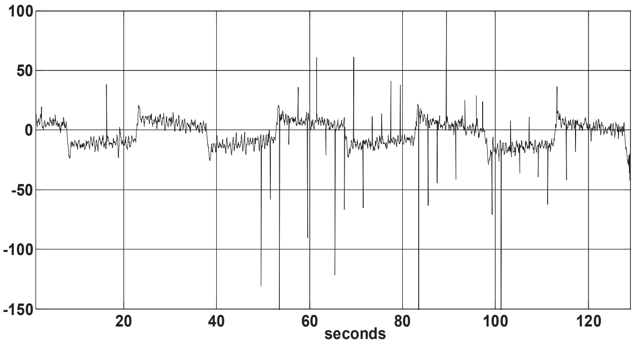


(a) Cart 3 position.

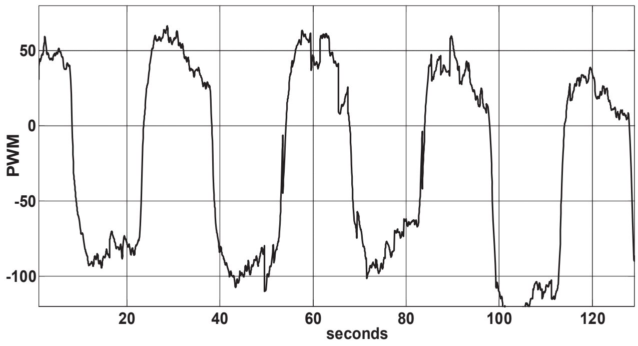


(b) The pendulum 3 angle θ .

Fig. 11. Experimental results for the cart 3, where the desired trajectory is a triagular wave. (a) The actual trajectory, desired trajectory and tracking error of the cart. (b) The pendulum angle θ .



(a) Control values of the cart 3 before Kalman filtering.



(b) Control values of the cart 3 after Kalman filtering.

Fig. 12. (a). Control values of the cart 3 before Kalman filtering. (b) Control values of the cart 3 after Kalman filtering.

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