A Quality-Based Fixed-Step Power Control Algorithm with Adaptive Target Threshold

Chi Wan Sung, Kin Kwong Leung, and Wing Shing Wong

***Abstract*—A simple adaptive fixed-step power control algorithm for mobile cellular systems is proposed. While most of the power control algorithms are based on the received signal-to-interference ratio (SIR) to adjust the transmitter power, our algorithm can be based on any generic quality-of-service (QoS) measure. Examples of such a measure include the SIR measure and the bit-error-rate (BER) measure. Furthermore, our algorithm does not require the knowledge of the exact relation between the QoS measure and the SIR. As long as the QoS measure changes monotonically with the SIR, our algorithm is proven to converge to a specified target region.**

***Index Terms*—Cellular mobile system, power control, quality-of-service (QoS) tracking.**

I. Introduction

POWER control is an important component of resource management in mobile communication systems. The basic goal is to adjust the transmitter power such that the signal-to-interference (SIR) performance of the system is optimized in some global sense. Earlier works [1], [9], [16] identified the problem as an eigenvalue problem for nonnegative matrices. It was shown that the dominant eigenvector provides the solution that maximizes the minimum SIR of all communication links. Thus, the optimal power vector can be obtained by computing the eigenvector. This approach is called *power balancing*. However, a centralized approach requires the information of all the link gains and the procedure of inverting a matrix. Due to the difficulty of estimating the link gains and the computational complexity, the centralized approach is not applicable in practice. Distributed versions which need only SIR information, which can be measured locally, have been developed [4], [10], [13], [17]. This power balancing approach forms the first paradigm of the power control problem in the literature.

A second paradigm was adopted by Foschini and Miljanic [3]. Instead of maximizing the SIR, the objective is to determine a power vector such that a given quality-of-service (QoS) requirement of all communication links can be satisfied. We call this approach *QoS tracking*. Moreover, Foschini and Miljanic also proposed a power control algorithm under which the power vector was shown to converge to an unique solution which minimizes the total power consumption. The convergence of the proposed algorithm under asynchronous operation was later proved in [8]. Afterwards, a framework based on this paradigm was developed in [14]. It can be applied to various interference models, which are commonly encountered in wireless communication systems. By identifying common properties of the interference constraints, the notion of *standard interference function* was introduced. In this paper, we use the same concept to generalize the common SIR measure. This second paradigm will be followed. Variations of the orginal power control problem under this paradigm can be found in [5], [7], [12], [15].

In most of the previous studies, the transmit power level can assume any value in a *continuous range*. However, in digital cellular systems or future PCS systems, the power level is quantized into discrete values. It is not clear how to apply those power control algorithms into a power-quantized system. For example, it may no longer be possible for the SIR of all users to converge to certain target values, as some algorithms (e.g., [3], [7], [8], [14]) assume. Hence, the convergence property of those algorithms may need to be re-examined. In our work, we will show that in a power-quantized system, the SIR target value should be changed into a SIR target region. In this paper, a fixed-step power control algorithm is proposed and it is proved to converge to the target region.

Another issue that we address in this paper is the information needed to drive the power control algorithm. Most power control algorithms proposed in the literature require knowledge of the received SIR. Some of them may require the estimate of the channel gains as well. The objective is to keep the SIR of all links above a pre-specified threshold. But in practice, the link gains and the SIR are often difficult to measure. Even if the SIR is measurable, the exact relationship to an actual quality measure, like the bit error rate (BER), is not known. What can be sure is only that the quality of a communication link depends strongly on the SIR. Thus, the desired performance is more appropriately expressed in terms of an actual quality meausre, instead of an SIR requirement. In [6], an algorithm based on BER was proposed. The SIR requirement was replaced by the BER requirement. The convergence of the algorithm was proved under the assumption that the BER is an exponential function of the SIR.

In this paper, we propose a distributed fixed-step algorithm, which is an extension of [11]. In the original version, the SIR of each user is required to converge to a target region, which is specified by two thresholds. It was proved that the algorithm converges as long as a feasible solution exists. A notable feature of the modification in this paper is that the upper threshold of the target region is now adjusted adaptively. It turns out that the power consumption can be reduced. More importantly, the power iteration is not necessarily based on the SIR measure.

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The authors are with the Department of Information Engineering, The Chinese University of Hong Kong, Shatin, N.T., Hong Kong (e-mail: cwsung@ie.cuhk.edu.hk; kkleung7@ie.cuhk.edu.hk; wswong@ie.cuhk.edu.hk).

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Any measure which indicates the link quality can be used instead. An example is the BER measure. Remarkably, there is no strict restriction on the quality measure. An explicit relationship between the quality measure and the SIR is not needed. As long as the quality measure changes monotonically with the SIR, the convergence of the algorithm is guaranteed. Thus the difficulty in estimating the SIR can be avoided.

II. System Model

Consider a cellular radio system. To each communication link, we allocate a pair of orthogonal channels (time slots or frequencies) for mobile-to-base (uplink) and base-to-mobile (downlink) communication. Since there is no interference between the uplink and the downlink channels, we consider power control for the uplink channel in this paper. However, the results can be applied to the downlink channels as well.

We focus on a set of cells in which a particular channel is used at a particular instant. Let M be the cardinality of this set. We ignore the effect of adjacent channel interference. Thus, mobile terminals which use other channels have no interference with our considered set.

Let P_i be the power transmitted by the ith mobile of our considered set. The link gain from mobile j to base station i is denoted by G_{ij}. The matrix $\mathbf{G} = \{G_{ij}\}$ is known as the uplink gain matrix. In reality, the link gains change constantly in time. Thus the link gain matrix $\mathbf{G}$ is actually a stochastic process. Assuming that the power control process is much faster than the rate of change of the channel, we consider a snapshot of the system so that $\mathbf{G}$ is treated as an $M \times M$ matrix of random variables.

In general, the SIR can be expressed in the following form:

$$\Gamma_i = \frac{P_i}{I_i(\mathbf{P})} \tag{1}$$

where $I_i(\mathbf{P})$ depends on the transmitted power of all terminals. We call it the *normalized interference function* and we assume that it satisfies the following three conditions:

1) *Positivity*: $I_i(\mathbf{P}) \geq \epsilon > 0$
2) *Monotonicity*: If $\mathbf{P} \geq \mathbf{P}'$, then $I_i(\mathbf{P}) \geq I_i(\mathbf{P}')$
3) *Scalability*: For all $\alpha > 1, \alpha I_i(\mathbf{P}) > I_i(\alpha \mathbf{P})$

where ϵ is an arbitrary, positive constant.

The above three conditions are the same as those in the definition of the notion *standard interference function* introduced in [14]. However, it needs to emphasize that our definition of interference function is slightly different from that in [14].

This generalized SIR measure can be applied to many different systems. For example, in a TDMA/FDMA system, $I_i(\mathbf{P})$ can be written as

$$I_i(\mathbf{P}) = \frac{1}{G_{ii}} \left(\sum_{j \neq i} G_{ij} P_j + \eta_i \right) \tag{2}$$

where η_i is the receiver noise. It is easy to verify that this expression of $I_i(\mathbf{P})$ satisfies the above three conditions.

Other examples to which this generalized SIR measure can be applied include systems which employ diversity techniques with maximal ratio combining [14] and CDMA systems using minimum mean square error (MMSE) multi-user detection [12].

The generalized SIR determines the quality of a communication link. However, in practice, the value of the SIR is difficult to estimate. In our power control algorithm, we do not use the SIR directly. Other QoS measure can be used instead. An example is the *bit error rate* (BER) measure, which can be obtained from the decoding algorithm. Our proposed algorithm does not depend on the particular measure used. The convergence of the algorithm is guaranteed provided that the quality measure is a strictly increasing function of the SIR. We let Λ_i be the quality measure of user i. In other words, we require that

$$\Gamma > \Gamma' \Rightarrow \Lambda(\Gamma) > \Lambda(\Gamma'). \tag{3}$$

For example, the BER measure is a decreasing function of the SIR. To use it in our power control algorithm, we can define Λ as the reciprocal of the BER.

Now, our goal is to find a feasible power vector such that $\Lambda_i \geq \alpha_i$ for all i, where α_i is the QoS requirement of user i.

III. Power Control Algorithm

Our algorithm is a modification of the algorithm in [11]. The original version is an algorithm based on SIR measurements. Our proposed version only indirectly relies on the SIR. The power level of each mobile terminal is adjusted according to a quality measure, Λ. Before we describe our proposed version, we state the original version first.

A. Fixed-Step Power Control Algorithm Based on SIR

Each mobile unit adjusts its transmit power $P_i^{(n+1)}$ in the $(n+1)$th step according to the following rules:

$$P_i^{(n+1)} = \begin{cases} \delta P_i^{(n)} & \text{if } \Gamma_i^{(n)} < \delta^{-1}\gamma_i \\ \delta^{-1} P_i^{(n)} & \text{if } \Gamma_i^{(n)} > \delta\gamma_i \\ P_i^{(n)} & \text{otherwise} \end{cases} \tag{4}$$

where $\delta > 1$.

In the original version, an SIR *target region* $[\delta^{-1}\gamma_i, \delta\gamma_i]$ is defined for each mobile. If the SIR is below the region, the base station will inform the mobile to raise the power to the next higher level. If the SIR is above the region, the power will be adjusted downwards by one level. Our new version basically follows the same idea. A target region for mobile i is specified by two QoS thresholds, α_i and β_i $(\alpha_i \leq \beta_i)$. α_i is the minimum quality requirement that link i must satisfy. The upper threshold, β_i, is needed such that the SIR will not be too high. If the SIR is exceedingly high, the transmit power may cause excessive interference to other cochannel users. Moreover, it is undesirable to waste battery power of the handset.

It should be noted that the target region is specified in terms of SIR in the orginial version. The upper threshold is $2\delta^{(\mathrm{dB})}$ dB greater than the lower threshold. (We use $x^{(\mathrm{dB})}$ to denote the decibel value of x, i.e., $x^{(\mathrm{dB})} = 10\log_{10} x$.) In our new version, the lower threshold is simply the QoS requirement. However, there is not enough information for us to determine the upper threshold since an exact relationship between the SIR and the link quality measure is not known. As a consequence, we propose to adjust the upper threshold adaptively. To guarantee the convergence of the algorithm, a probabilistic element is introduced. The algorithm is stated below.

B. Adaptive Fixed-Step Power Control Algorithm Based on Link Quality Measure

Each mobile unit adjusts its transmit power, $P_i^{(n+1)}$, and the upper threshold, $\beta_i^{(n)}$ in the $(n+1)$th step according to the following rules:

1) If $\Lambda_i^{(n)} < \alpha_i$,

$$P_i^{(n+1)} = \delta P_i^{(n)}$$

If $\Lambda_i^{(n+1)} > \beta_i^{(n)}$,

let $\beta_i^{(n+1)} = \Lambda_i^{(n+1)}$

end

end.

2) If $\Lambda_i^{(n)} > \beta_i^{(n)}$,

$$P_i^{(n+1)} = \begin{cases} \delta^{-1} P_i^{(n)} & \text{with probability } q \\ P_i^{(n)} & \text{with probability } 1-q \end{cases}$$

If $\Lambda_i^{(n+1)} < \alpha_i$,

let $\beta_i^{(n+1)} = \Lambda_i^{(n)}$

end

end.

3) Otherwise,

$$P_i^{(n+1)} = P_i^{(n)}$$

end

Note that the upper threshold, $\beta_i^{(0)}$, is initialized to the lower threshold, α_i for all i.

Fig. 1 shows the two cases where the upper threshold, β_i will be adjusted. Fig. 1(a) and (b) corresponds to the cases in rule 1 and 2, respectively. Note that $\beta_i^{(n)}$ is a monotonic increasing sequence.

Another feature of the algorithm is that it is probabilistic. If the QoS measure is greater than the upper threshold, the mobile terminal decreases its power with probability q. With probability $1-q$, it keeps its power unchanged. With this probabilitic rule, the algorithm is guaranteed to be stable and the QoS measure can be kept above the requirement, α_i, for all link i. The proof will be presented in the next section.

From now on, we assume that $\alpha_i = \Lambda(\delta^{-1}\gamma_i)$. Since the change of the SIR between two consecutive power control iterations is at most $2\delta^{(\text{dB})}$ dB, $\beta_i^{(n)}$ is less than $\Lambda(\delta\gamma_i)$ for all n. In general, the target region, in terms of SIR, is smaller than that in the original version.

IV. Quantization of Power Level

In our algorithm, the power level is quantized. The difference between two consecutive levels is equal to $\delta^{(\text{dB})}$ dB. The SIR of each user is required to converge to a target region, instead of a target value. In [11], the target region is $2\delta^{(\text{dB})}$ dB wide. It

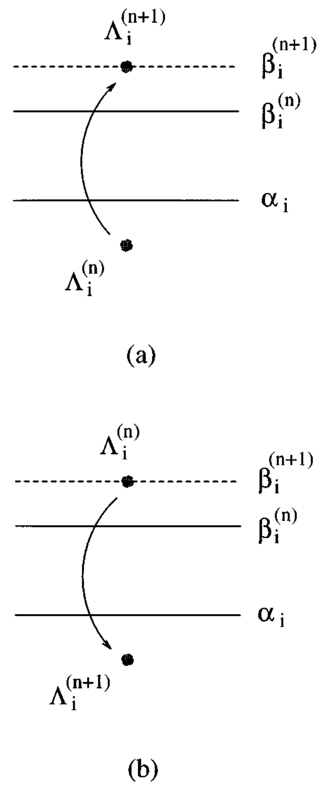

Fig. 1. The adjustment of the upper threshold, β_i at the $n+1$ iteration. (a) Corresponds to the case in rule 1. (b) Corresponds to the case in rule 2.

was proved that a feasible solution exists if the SIR of each user can converge to the mid-value of the target region in a system without power quantization.

In this section, the result proved in [11] will be refined. We will show that a feasible solution exists even if we narrow the width of the target region to $\delta^{(\text{dB})}$ dB. We state our result as follows.

Theorem 1: If there exists a power vector $\mathbf{P}^*$ such that $\Gamma_i(\mathbf{P}^*) = \gamma_i$ for all i, then there exists a quantized power vector $\hat{\mathbf{P}}$ such that $\delta^{-1}\gamma_i \le \Gamma_i(\hat{\mathbf{P}}) \le \gamma_i$ for all i.

Proof: Given P_i^*, we can always find one and only one quantized power level $\hat{P}_i^*$ such that $\delta^{-1/2}P_i^* \le \hat{P}_i^* < \delta^{1/2}P_i^*$. Assume that P_i^* is quantized to $\hat{P}_i^*$. Let $\hat{\mathbf{P}}^*$ be the quantized power vector corresponding to the given vector $\mathbf{P}^*$

$$\Gamma_i(\hat{\mathbf{P}}^*) = \frac{\hat{P}_i^*}{I_i(\hat{\mathbf{P}}^*)} \tag{5}$$

$$\ge \frac{\delta^{-1/2}P_i^*}{I_i(\hat{\mathbf{P}}^*)} \tag{6}$$

$$\ge \frac{\delta^{-1/2}P_i^*}{I_i(\delta^{1/2}\mathbf{P}^*)} \tag{7}$$

$$\ge \frac{\delta^{-1/2}P_i^*}{\delta^{1/2}I_i(\mathbf{P}^*)} \tag{8}$$

$$= \delta^{-1}\Gamma_i(\mathbf{P}^*) \tag{9}$$

$$= \delta^{-1}\gamma_i \tag{10}$$

where the second inequality follows from monotonicity and the third inequality follows from the scalability of the interference function.

Similarly, we can show that

$$\Gamma_i(\hat{\mathbf{P}}^*) \leq \delta\gamma_i. \tag{11}$$

Hence, the given condition implies that there exists a quantized power vector, $\hat{\mathbf{P}}^*$, such that

$$\delta^{-1}\gamma_i \leq \Gamma_i(\hat{\mathbf{P}}^*) \leq \delta\gamma_i \tag{12}$$

for all i.

Now we assume that $\hat{\mathbf{P}}^{(0)} = \hat{\mathbf{P}}^*$ and we consider the following iterative process.

If $\Gamma_i^{(n)} > \gamma_i$, then $\hat{P}_i^{(n+1)} = \delta^{-1}\hat{P}_i^{(n)}$. Otherwise, $\hat{P}_i^{(n+1)} = \hat{P}_i^{(n)}$.

Note that the power level of each user is monotonic decreasing, i.e., $\hat{\mathbf{P}}^{(n+1)} \leq \hat{\mathbf{P}}^{(n)}$. If $\Gamma_i^{(n)} \leq \gamma_i$, then $\Gamma_i^{(n+1)} \geq \Gamma_i^{(n)}$ because $I_i(\hat{\mathbf{P}}^{(n+1)}) \leq I_i(\hat{\mathbf{P}}^{(n)})$. If $\Gamma_i^{(n)} > \gamma_i$, then

$$\Gamma_i^{(n+1)} = \frac{\delta^{-1}\hat{P}_i^{(n)}}{I_i\left(\hat{\mathbf{P}}^{(n+1)}\right)} \tag{13}$$

$$\geq \frac{\delta^{-1}\hat{P}_i^{(n)}}{I_i\left(\hat{\mathbf{P}}^{(n)}\right)} \tag{14}$$

$$= \delta^{-1}\Gamma_i^{(n)} \tag{15}$$

$$\geq \delta^{-1}\gamma_i. \tag{16}$$

Hence, we have $\Gamma_i^{(n)} \geq \delta^{-1}\gamma_i$ for all i and all iteration n.

Now assume that there does not exist an interger N such that $\Gamma_i^{(n)} \leq \gamma_i$ for all $n \geq N$. As a consequence, $P_i^{(n)}$ tends to zero. Since $I_i(\mathbf{P}) \geq \epsilon, \Gamma_i^{(n)}$ also goes to zero, which contradicts with the fact that $\Gamma_i^{(n)} \geq \delta^{-1}\gamma_i$. Hence, there exists an integer N such that $\Gamma_i^{(n)} \leq \gamma_i$ for all i and all $n \geq N$. The power vector $\hat{\mathbf{P}}^{(N)}$ is thus our desired quantized power vector. ■

Assume that the SIR requirement can be satisfied if the power vector is not restricted to discrete values. Let us focus on the restriction due to the power quantization. Theorem 1 in [11] states that a feasible solution can be found if the width of the target region is $2\delta^{(\mathrm{dB})}$ dB. The above theorem refines the previous observation. It states that a feasible solution exists even if we reduce the width of the target region to $\delta^{(\mathrm{dB})}$ dB. (There may be many feasible solutions.) This gives an additional advantage to our algorithm. In our algorithm, the width of the target region is increased monotonically from zero until a feasible solution is found. Although there is no guarantee that the resulting solution will fall into a region of width less than or equal to $\delta^{(\mathrm{dB})}$ dB, it can be imagined that the SIR tends to converge to a narrower region than that using the orginal algorithm. The advantage is that the power consumption can be reduced. This point will be justified by simulation in a later section.

V. Convergence Property

It was shown in [11] that the existence of a power vector $\mathbf{P}$ such that $\Gamma_i = \gamma_i$ for all i implies that a quantized power vector $\hat{\mathbf{P}}$ can be found such that $\delta^{-1}\gamma \leq \Gamma_i(\hat{\mathbf{P}}) \leq \delta\gamma$ for all i. A direct consequence of this result is as follows.

Theorem 2: If there exists a power vector $\mathbf{P}^*$ such that $\Lambda_i(\Gamma_i(\mathbf{P}^*)) = \Lambda(\gamma_i)$ for all i, then there exists a quantized power vector $\hat{\mathbf{P}}$ such that $\Lambda(\delta^{-1}\gamma_i) \leq \Lambda(\Gamma_i(\hat{\mathbf{P}})) \leq \Lambda(\delta\gamma_i)$ for all i.

Now we assume that a power-quantized vector $\hat{\mathbf{P}}$ such that $\Lambda_i \geq \alpha_i$ for all i can be found. By Lemma 1 and 2 (in Appendix A), there exists an upper bound and a lower bound for the power level, $P_i^{(n)}$, of each mobile terminal. We denote them by U_i and L_i, respectively. Let $U = \max U_i$ and $L = \min L_i$.

Let w be the number of quantization levels between U and L.

$$w = \log_\delta \frac{U}{L} + 1. \tag{17}$$

Now we proceed to prove the convergence of the fixed-step algorithm. Our results are summarized in two propositions and one theorem. Proposition 1 states that the upper threshold, $\beta_i^{(n)}$, will stop increasing for sufficiently large n. Proposition 2 states that a feasbile solution, if exists, will be found by the algorithm. Theorem 3 states that the algorithm will converge to a fixed-point which is within the target region with probability 1.

Proposition 1: For each mobile, there exists $n_0(i)$ such that the upper threshold, $\beta_i^{(n)}$, is equal to a constant value, β_i^*, for $n \geq n_0(i)$.

Proof: Totally, there are M users in the system. The power level of each user can assume at most w different values. Therefore, there are at most w^M different power vectors during the power iteration process. In consequence, there are at most w^M different SIR values which can be achieved by each user. According to the definition of $\beta_i^{(n)}$, for $n > 0$, it can be expressed in the form

$$\beta_i^{(n)} = \Lambda_i(\Gamma_i) \tag{18}$$

for some Γ_i.

Therefore, $\beta_i^{(n)}$ can assume only a finite number of values. Since $\beta_i^{(n)}$ is an increasing sequence and is upper bounded by $\Lambda(\delta\gamma_i)$, it must remain constant when n is sufficiently large. ■

Proposition 2: If there exists $\hat{\mathbf{P}}$ such that $\Lambda_i \geq \alpha_i$ for all i, then given any $n_0 \geq 0$, with probability 1 there exists $N \geq n_0$ such that $\Lambda_i^{(N)} \geq \alpha_i$ for all i.

Proof: First of all, we consider the scenario that no mobile terminal decreases its power level during an interval of $M(w-1)+1$ iterations. In such an interval, the power levels of all the terminals are monotonic increasing (a sequence $\{x_n\}$ is monotonic increasing if $x_{n+1} \geq x_n$). We call it an I-interval. Now assume that the time axis is partitioned into intervals. Let r be the probability that a given interval is an I-interval. We would like to prove that r is strictly greater than 0. Before doing that, we first consider the probability where no one decreases its power at m consecutive iteration stages. We denote this probability by r_m. Now we show that r_1 must be greater than 0. Assume that at the iteration concerned, there are x mobiles greater than the corresponding upper threshold.

$$r_1 = (1-q)^x \tag{19}$$

$$\geq (1-q)^M \tag{20}$$

$$> 0. \tag{21}$$

Then

$$r = r_{M(w-1)+1} \geq ((1-q)^M)^{M(w-1)+1} > 0. \tag{22}$$

Now we consider n consecutive intervals, each of which has $M(w-1)+1$ iteration stages. Let s_n be the probability that none of the n intervals is an I-interval. Then we have

$$s_n = (1-r)^n. \tag{23}$$

When n tends to infinity,

$$s_n \to 0. \tag{24}$$

Therefore, given any $n_0 \geq 0$, with probability 1 we can find an I-interval after iteration n_0.

Assume that an I-interval occurs. Since there are only w quantized power levels for each mobile, each mobile can increase its power at most $(w-1)$ times. Therefore, among the $M(w-1)+1$ iteration stages, there is at least 1 stage at which no mobile changes its power. It implies that $\Lambda_i \geq \alpha_i$ for all i at that particular stage. ■

Theorem 3: If there exists a power-quantized vector $\hat{\mathbf{P}}$ such that $\Lambda_i \geq \alpha_i$ for all i, then the fixed-step power control algorithm converges with probability 1 to a fixed point $\mathbf{P}^*$ where $\alpha_i \leq \Lambda_i^* \leq \beta_i^* \leq \Lambda(\delta\gamma_i)$ for all i.

Proof: By Proposition 1, there exists n_0 such that $\beta_i^{(n)} = \beta_i^*$ for all i and $n \geq n_0$. By Proposition 2, given n_0, with probability 1, there exists $N \geq n_0$ such that $\Lambda_i^{(N)} \geq \alpha_i$ for all i. Note that this condition implies that no mobile increases its power at iteration $N+1$.

Consider a mobile i whose quality measure is within the target region at iteration stage N, i.e., $\alpha_i \leq \Lambda_i^{(N)} \leq \beta_i^*$. Since the power level of that mobile will not change at iteration $N+1$, and no other mobiles increases its power level, we have $\Lambda_i^{(N+1)} \geq \Lambda_i^{(N)} \geq \alpha_i$.

Next, consider the other mobiles. If $\Lambda_i^{(N)} \geq \beta_i^*$, mobile i may decrease its power level or keep the power unchanged. If it keeps its power unchanged, obviously we have $\Lambda_i^{(N+1)} \geq \Lambda_i^{(N)} \geq \alpha_i$. If it decreases its power level, $\Lambda_i^{(N+1)}$ will decrease. However, we still have $\Lambda_i^{(N+1)} \geq \alpha_i$, for otherwise $\beta_i^{(N+1)}$ will be greater than $\beta_i^{(N)}$ which violates the assumption that $\beta_i^{(n)} = \beta_i^*$ for $n \geq n_0$. Hence, we conclude that $\Lambda_i^{(n)} \geq \alpha_i$ for all $n \geq N$. As a direct consequence, the power level is monotonic decreasing from iteration N onwards.

Now it remains to prove that there exists $n \geq N$ such that $\Lambda_i^{(n)} \leq \beta_i^*$ for all i. Assume that at iteration N, there exists i such that $\Lambda_i^{(N)} > \beta_i^*$. It is obvious that the probability that $\mathbf{P}^{(n)} = \mathbf{P}^{(N)}$ for all $n \geq N$ is zero. So with probability 1, we can find $n \geq N$ such that $\mathbf{P}^{(n)} < \mathbf{P}^{(N)}$. If there are still some mobiles outside the target region, by the same argument, the power vector will strictly decrease. Since the power vector has a lower bound and the power level decreases in discrete steps, $\Lambda_i^{(n)}$ will ultimately fall within the target region for sufficiently large n. ■

VI. Numerical Results

Some simulation studies on the fixed-step power control algorithm were conducted, assuming a standard hexagonal cellular layout with sixteen cochannel cells (see Fig. 2) [13]. The geographical location of the cells corresponds to a reuse pattern of seven. We approximate each hexagonal cell by a circular cell of the same area. Within each cell, there is a mobile terminal communicating with the base station. The location of each mobile terminal is generated uniformly inside the cell. The link gain G_{ij} is defined as

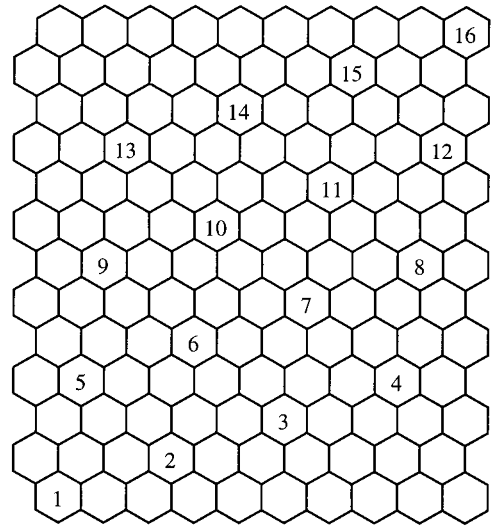

Fig. 2. Layout of interfering cells in the numerical study.

$$G_{ij} = \frac{A_{ij}}{d_{ij}^4}$$

where d_{ij} is the distance between the ith base station and the jth mobile terminal and A_{ij} is the corresponding attenuation factor. In this study, we consider only log-normal fading. Hence, we assume A_{ij} is log-normal distributed with mean 0 dB and standard deviation 6 dB for all i and j. The receiver noise η_i is assumed 10^{-6} for all i.

First of all, we investigate the effect of q on the performance of our algorithm. We let the step size, $\delta^{(\mathrm{dB})}$, be 1 dB. For simplification and ease of comparison with other algorithms, we assume that our QoS measure, Λ, is the SIR measure. For acceptable link quality, we assume that an SIR of 15 dB is required. To ensure that the SIR of each user be greater than or equal to 15 dB, we let the lower threshold, $\alpha_i^{(\mathrm{dB})}$, be 15 dB. Thus, the SIR should converge to a range between 15 and 17 dB.

In Fig. 3, we show a typical result for the evolution of the maximum and minimum SIR of the two cases: $q = 0.95$ and $q = 0.5$. It can be seen that for large q, the convergence is faster. However, if we consider the number of iterations for all the mobiles to meet the QoS requirement (i.e., the QoS measure of each mobile above its lower threshold, $\Lambda_i \geq \alpha_i$ for all i), the two cases are roughly the same. Thus, the value of q has limited effect on the link quality.

On the other hand, the value of q does affect the power consumption. Fig. 4 shows the average transmit power for different values of q. We find that the transmit power decreases when q increases. The reason is as follows. In our discrete power control model, there are many feasible solutions at which the SIR of each user falls within the target region. If q is small, a mobile whose SIR is greater than the upper threshold remains constant most of the time. As a result, those mobiles whose SIR

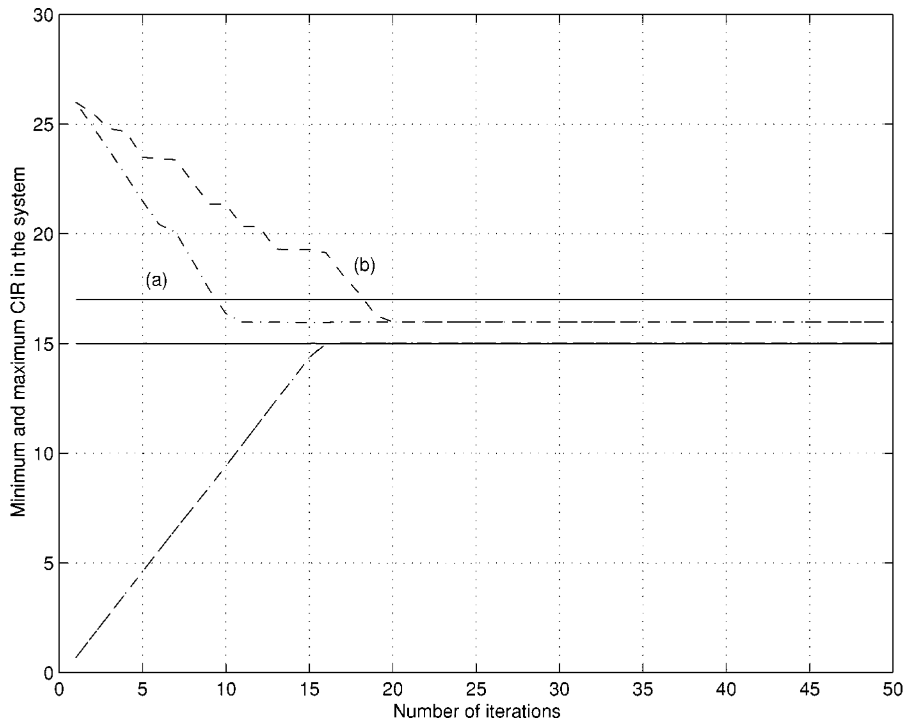

Fig. 3. A typical case: minimum and maximum SIR of the fixed-step algorithm (a) $q = 0.95$ (b) $q = 0.5$ (step size $\delta^{(\mathrm{dB})} = 1$ dB).

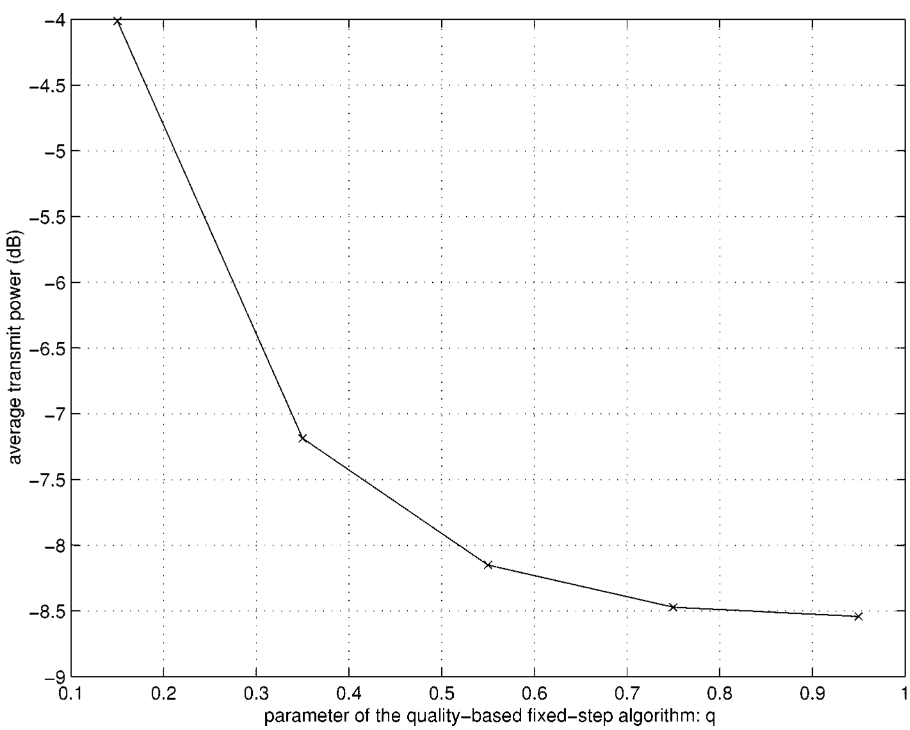

Fig. 4. Average transmit power of the quality-based fixed-step algorithm for different values of q (step size $\delta^{(\mathrm{dB})} = 1$ dB).

is lower than the lower threshold need to increase their power many times in order to meet the quality requirement. Then it tends to converge to a solution at which the power of each mobile is higher. However, if q is large, those mobiles whose SIR is high will decrease their power, thus reducing the interference to others. The power levels of the mobiles thus converge to values which are smaller in magnitude. Therefore, it is preferable to choose a larger value of q. However, if it is equal to one, the algorithm may be unstable for some situations (see Appendix B). From now on, we assume that $q = 0.95$.

To investigate the performance of our proposed algorithm, we compare it with Foschini–Miljanic algorithm [3], which is well known in the literature. Foschini–Miljanic algorithm is an SIR-based algorithm, which allows the power to take any positive value. Under this algorithm, the power vector is shown to converge to an unique solution which minimizes the total

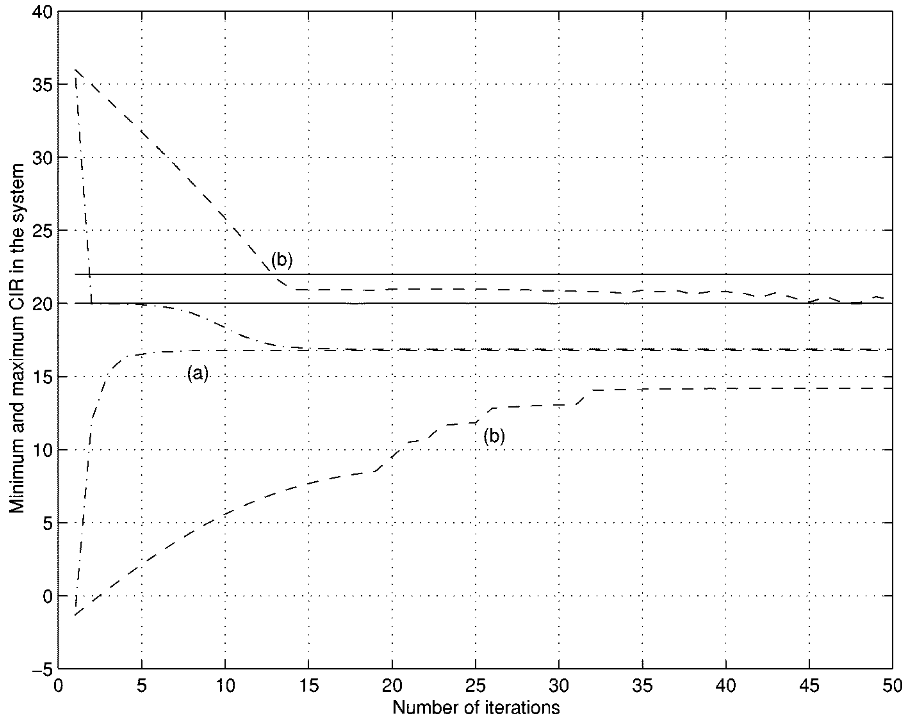

Fig. 5. A typical case: minimum and maximum SIR of (a) the Foschini–Miljanic algorithm and (b) the fixed-step algorithm (step size $\delta^{(\mathrm{dB})} = 1$ dB, $q = 0.95$).

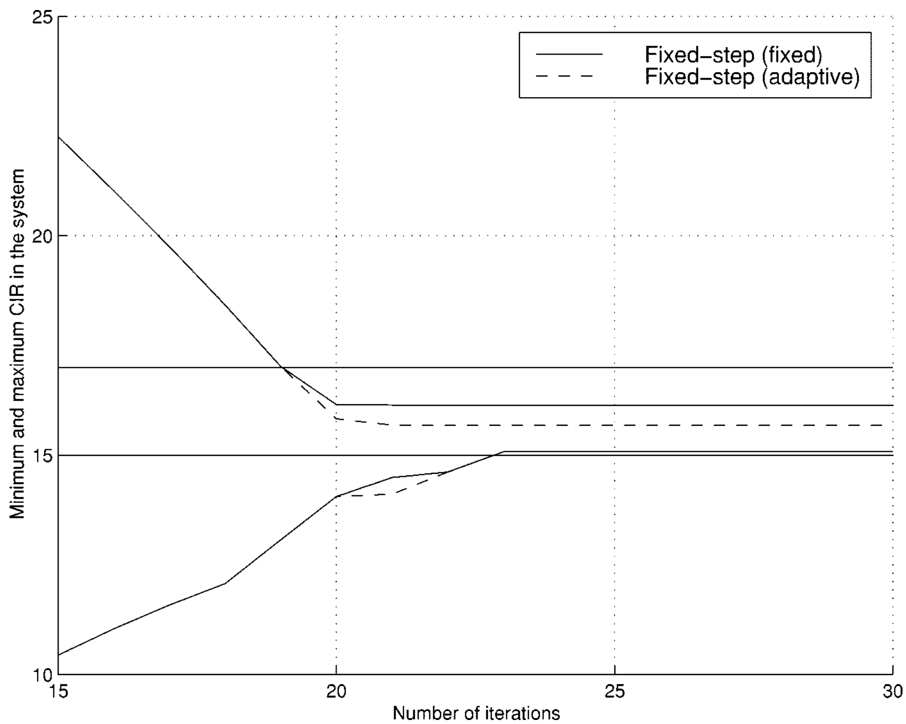

Fig. 6. A typical case: minimum and maximum SIR of the two versions of the fixed-step algorithm (step size $\delta^{(\mathrm{dB})} = 1$ dB, (adaptive) $q = 0.95$).

transmit power for a given SIR requirement [3]. We state the algorithm as follows:

$$P_i^{(n+1)} = \frac{\gamma_i}{\Gamma_i^{(n)}} P_i^{(n)}. \tag{25}$$

We first compare the convergence rate of the two algorithms. As before, we assume that the SIR should be greater than or equal to 15 dB. Thus we set $\gamma_i^{(\mathrm{dB})}$ to 15 dB in Foschini–Miljanic algorithm. For the fixed-step algorithm, we let the step size and the lower threshold be 1 dB and 15 dB, respectively. Thus the SIR should converge to a range between 15 and 17 dB. Fig. 5 shows a typical plot for the maximum and the minimum SIR among the sixteen users. The target region is represented by the pair of solid lines. It can be seen that Foschini–Miljanic algorithm converges faster than the

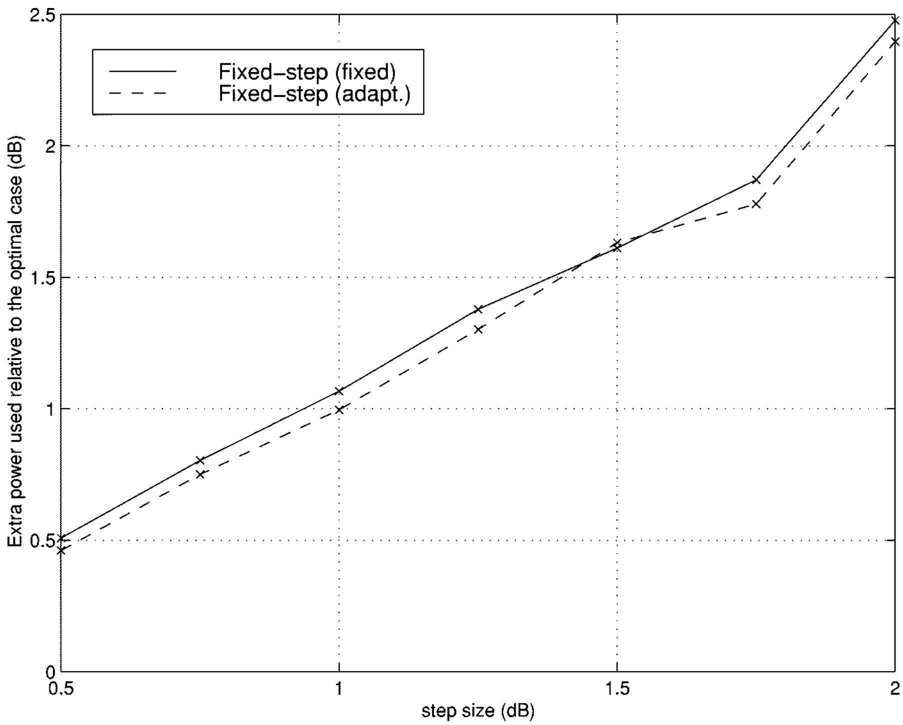

Fig. 7. Extra power used by the fixed-step algorithm with different initial power (step size $\delta^{(\mathrm{dB})} = 1$ dB).

fixed-step algorithm in terms of number of iterations. However, it should be noted that the fixed-step algorithm uses only two bits for each control command. If the bandwidth of the control channel is limited, the fixed-step algorithm can have more iterations per unit of time. The convergence rate of the two algorithms will become closer.

If we compare our proposed fixed-step algorithm with the original version with fixed target threshold, we find that their convergence rates are roughly the same. A typical plot is shown in Fig. 6. On the other hand, the adaptive version forces the SIRs to converge to a narrower region. It means that the resulting SIR is closer to the threshold, i.e., 15 dB. An important consequence is that the power consumption can be reduced. This phenomenon will be investigated explicitly in the next experiment.

In the fixed-step algorithm, the transmit power is quantized into discrete levels. Due to this restriction, we can only require the SIR to fall into a target region, instead of a target value. As a consequence, the solution is not unqiue and more power may be used. As the solution obtained by Foschini–Miljanic algorithm is optimal in the sense of minimizing the power consumption, we use it as a reference point. The solution yielded by the two versions of the fixed-step algorithm will be compared with it.

Fig. 7 shows the extra power used by the two fixed-step algorithms. Since there are many feasible solutions, the solution to which a discrete algorithm converges depends on the initial power vector. It can be seen that the higher the initial power, the more power will be used. This result suggests that a mobile should use a low power initially and then progressively increase it until the QoS requirement is met. On the other hand, if we compare the two fixed-step algorithms, we can see that the one with adaptive threshold uses less power than the other. This result is in agreement with the fact that the target region of the adaptive algorithm is smaller than the other one.

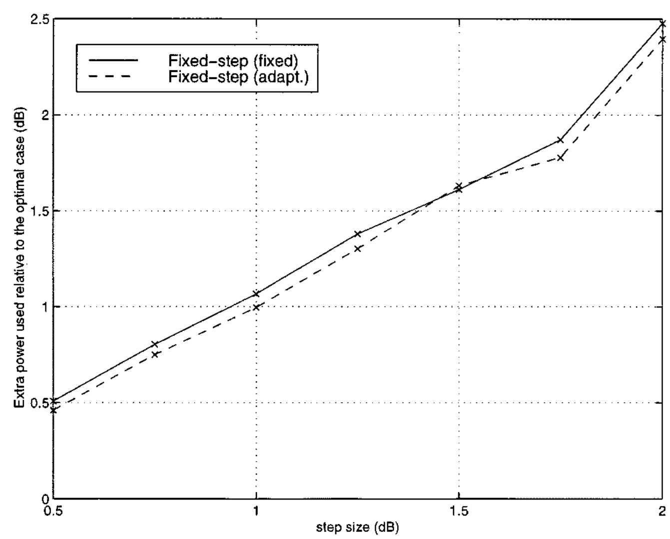

Fig. 8. Extra power used by the fixed-step algorithm with different step sizes (inital power $= -20$ dB).

Now assume that the initial power is set low enough. As seen from Fig. 7, the extra power used by the fixed-step algorithm is about 1 dB, which equals the step size $\delta^{(\mathrm{dB})}$. This prompts us to investigate the effect of the step size on the power consumption.

Fig. 8 shows the extra power used by the fixed-step algorithms with different step sizes. For both versions, the average power increases when the step size increases. It is because a large step size implies that the SIR falls into a large target region. Thus more power is consumed.

VII. Conclusion

In this paper, we have improved the fixed-step algorithm originally presented in [11]. With this new algorithm, the power control mechanism can be driven by any QoS measure, provided that the measure is a monotonic function of the SIR. This avoids the difficulty of estimating the SIR accurately in real time. For example, the BER, which is readily available from the decoding algorithm, can be used.

As for the original algorithm, we have proved the convergence of this new algorithm. Since the upper threshold is adaptively adjusted in this new version, the convergence region is usually smaller than the original one. An important consequence is that the power consumption can be reduced, which prolongs the battery life of the mobile handset.

Another notable feature of this new algorithm is that a probabilistic element is introduced. This is necessary to ensure the stablity of the algorithm. In our simulation, we show that a large value of q reduces the power consumption. However, in the extreme case when $q = 1$, the algorithm may oscillate in some situations. An example is given in Appendix B. If such a situation occurs, a small value of q is needed to break the oscillating pattern quickly. However, the occurrence of such an oscillating pattern seems rare. Likewise, we conjecture that such an oscillating pattern will not occur if all the users update their powers at different times. (In fact, such an asynchronous situation is more realistic as it occurs with probability one if we assume that the arrival instant of each user is uniformly distributed along the time axis.) Therefore, we suggest to use a large value of q in practice. Moreover, further research on the convergence of the algorithm with an asynchronous model is needed.

Appendix A

Lemma 1: If there exists a power-quantized vector $\hat{\mathbf{P}}$ such that $\Lambda_i \geq \alpha_i$ for all i, then for each mobile terminal, its power level at each iteration stage under the fixed-step power control algorithm is upper bounded by a constant which depends on the gain matrix and the initial power vector.

Proof: Let $\mathbf{P}^{(0)}$ be the initial power vector. $P_i^{(0)}$ differs from $\hat{P}_i$ by a multiple of $\delta^{(\mathrm{dB})}$ dB, i.e.,

$$P_i^{(0)} = \hat{P}_i \delta^{a(i,0)} \tag{26}$$

where $a(i,0)$ is an integer. In general, we define $a(i,n)$ by

$$P_i^{(n)} = \hat{P}_i \delta^{a(i,n)}. \tag{27}$$

Note that $a(i,n)$ is an integer and $|a(i,n+1) - a(i,n)| \leq 1$.

Now, define

$$K(n) = \max_i \{a(i,n), 0\}. \tag{28}$$

For mobile i where $a(i,n) = K(n)$, we have

$$\Gamma_i^{(n)} = \frac{P_i^{(n)}}{I_i(\mathbf{P})} \tag{29}$$

$$\geq \frac{\hat{P}_i \delta^{K(n)}}{I_i\left(\hat{\mathbf{P}}\delta^{K(n)}\right)} \tag{30}$$

$$\geq \frac{\hat{P}_i \delta^{K(n)}}{I_i(\hat{\mathbf{P}})\delta^{K(n)}} \tag{31}$$

$$= \Gamma_i(\hat{\mathbf{P}}) \tag{32}$$

$$\geq \delta^{-1}\gamma_i \tag{33}$$

where the first inequality follows from the monotonicity of $I(\cdot)$ and the second from the scalability. Therefore, mobile i will not increase its power at iteration $n+1$. In other words, $a(i,n+1) \leq a(i,n) = K(n)$.

For mobile i where $a(i,n) < K(n)$, we have

$$a(i,n+1) \leq a(i,n) + 1 \leq K(n). \tag{34}$$

Hence, $K(n)$ is a nonincreasing sequence. As a result, for every mobile i, we have

$$P_i^{(n)} \leq \hat{P}_i \delta^{K(0)}. \tag{35}$$

■

Lemma 2: For each mobile terminal, its power level at each iteration stage under the fixed-step power control algorithm is lower bounded by a constant which depends on the gain matrix and the initial power vector.

Proof: Since the receiver noise is nonzero, it is obvious that there exists a power-quantized vector $\hat{\mathbf{P}}$ such that $\Gamma_i(\hat{\mathbf{P}}) \leq \delta^{-1}\gamma_i$ for all i.

Similar to the previous lemma, we define $a(i,n)$ by

$$P_i^{(n)} = \hat{P}_i \delta^{a(i,n)} \tag{36}$$

and

$$k(n) = \max_i \{-a(i,n), 0\}. \tag{37}$$

Following the same lines of reasoning, the mobiles which achieve the maximum value $k(n)$ cannot decrease their power at the next iteration because

$$\Gamma_i^{(n)} \leq \Gamma_i(\hat{\mathbf{P}}) \leq \delta^{-1}\gamma_i$$

For other mobiles, we have

$$-a(i,n+1) \leq -a(i,n) + 1 \leq K(n) \tag{38}$$

Therefore, $k(n)$ is a nonincreasing sequence. For every mobile i, we have

$$P_i^{(n)} \geq \hat{P}_i \delta^{-k(0)} \tag{39}$$

■

Appendix B

In our power control algorithm, we introduce a probabilistic element, $q < 1$, which governs the decrement of the power levels. In this Appendix, we cosntruct an example which shows the instability of the algorithm when $q = 1$.

TABLE I
AN EXAMPLE: POWER ITERATIONS AND THE CORRESPONDING SIR

Iteration	P_1	P_2	P_3	P_4	Γ_1	Γ_2	Γ_3	Γ_4
n	4	6	4	6	1.33	3	2	2
$n+1$	6	4	4	6	2	2	1.33	3
$n+2$	6	4	6	4	3	1.33	2	2
$n+3$	4	6	6	4	2	2	3	1.33
$n+4$	4	6	4	6	1.33	3	2	2

We assume that there are four mobiles. The link gain matrix $\mathbf{G}$ is given by the following. Note that it is highly asymmetric

$$\mathbf{G} = \begin{bmatrix} 1 & 0 & 0 & 0.5 \\ 0 & 1 & 0.5 & 0 \\ 0.5 & 0 & 1 & 0 \\ 0 & 0.5 & 0 & 1 \end{bmatrix}. \tag{40}$$

Assume that the step size, δ, is equal to 1.5 (linear scale) and γ_i is equal to 2.1 (linear scale) for $i = 1, 2, 3$ and 4. Hence, all the mobiles have a common lower threshold, α, which equals $\delta^{-1}\gamma_i$ or 1.4. The noise term, η_i, is assumed very small and is neglected in our calculations.

Assume that the power control algorithm has iterated a certain number of times and at the n-th iteration, the upper threshold, $\beta_i^{(n)}$, is equal to 2.1 for $i = 1, 2, 3$, and 4. The power vector, $\mathbf{P}^{(n)}$, is $[4\,6\,4\,6]$.

It is easy to see that when the power level of the mobiles are all equal, say $\mathbf{P} = [4\,4\,4\,4]$, the SIR of all users are equal to 2, which fall within the target region. However, we will show that in this example, the power vector does not converge.

The iterative procedure is shown in Table I. It can be checked that the upper threshold of each user, β_i, remains unchanged from iteration n to $n+4$. The power vector, $\mathbf{P}^{(n)}$, is the same as $\mathbf{P}^{(n+4)}$. Hence, the whole system repeats. In this example, the power vector does not converge and is trapped in the cycle.

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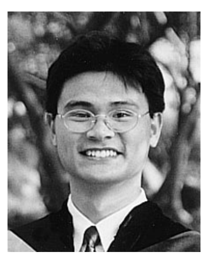
Chi Wan Sung received the B.Eng., M.Phil. and Ph.D. degrees in information engineering in 1993, 1995, and 1998, respectively, all from the Chinese University of Hong Kong, Shatin, Hong Kong.

After graduation in 1998, he became a Visiting Assistant Professor in the Department of Information Engineering, Chinese University of Hong Kong. Currently, he is Postdoctoral Fellow in the same department. His research interest is on resource management in wireless communication systems.



Kin Kwong Leung received the B.Eng. and M.Phil. in information engineering from the Chinese University of Hong Kong, Shatin, Hong Kong, in 1997 and 1999, respectively. He is currently working towards the Ph.D. degree in the Department of Information Engineering, the Chinese University of Hong Kong.

Wing Shing Wong received the combined M.A.B.A. degree in 1976 from Yale University, New Haven, CT, and the M.S. and Ph.D. degrees in 1978 and 1980, respectively, both from Harvard University, Cambridge, MA.

He joined AT&T Bell Laboratories in 1982 and was promoted to Supervisor in 1987. He joined the Information Engineering Department, Chinese University of Hong Kong, Shatin, Hong Kong, in 1992, where he is the Chairman and Professor of Information Engineering. His current research interests include mobile communication systems, analog computing, nonlinear filtering, performance analysis, and information issues in estimation and control. He is leading a development effort to prototype mobile integrated services on DECT.

Dr. Wong was an Associate Editor of the IEEE TRANSACTIONS ON AUTOMATIC CONTROL for four years.