Networks of Open Interaction

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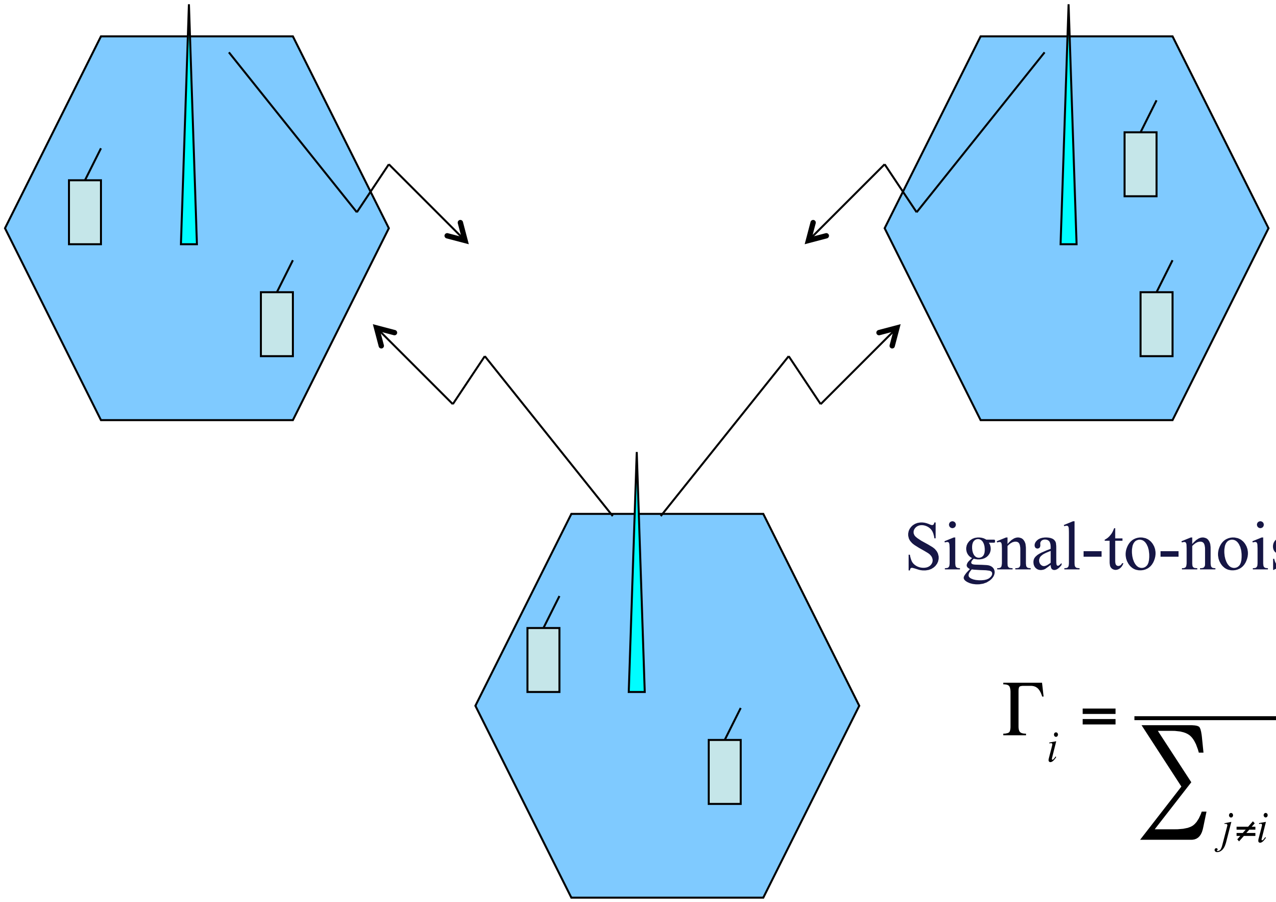
Talk Outline

- What is an NOI?
  - Examples for motivation
- Fundamental issues
  - Choice-based objectives
  - Control-communication interplay
  - Control under resource contention
- Concluding remarks

Examples for motivation

1. The power control problem



Signal-to-noise ratio for user i

$$\Gamma_i = \frac{P_i}{\sum_{j \neq i} Z_{ij} P_j + \eta_i}$$

The power vector $(P_1, P_2, \ldots, P_n)$ is non-negative

$\mathbf{Z} = \left[Z_{ij} \right]$, the normalized gain matrix is positive

The QoS tracking model

- Given signal-to-noise QoS objectives, γ_i, find distributed algorithms to achieve

$$\Gamma_i \geq \gamma_i$$

- Foschini-Miljanic Algorithm:

$$P_i^{(k+1)} = \frac{\gamma_i}{\Gamma_i^{(k)}} P_i^{(k)}$$

- It is known that the power vector, $\mathbf{P}^{(k)}$, converges to a feasible solution if a feasible solution exists.
- Setting of information-based control

Distributed algorithms

- Tri-State Algorithm (Sung-Wong):

$$P_i^{(k+1)} = \begin{cases} \delta P_i^{(k)} & \Gamma_i^{(k)} < \delta^{-1}\gamma_i, \\ \delta^{-1} P_i^{(k)} & \Gamma_i^{(k)} > \delta\gamma_i, \\ P_i^{(k)} & \text{otherwise,} \end{cases} \qquad \delta > 1.$$

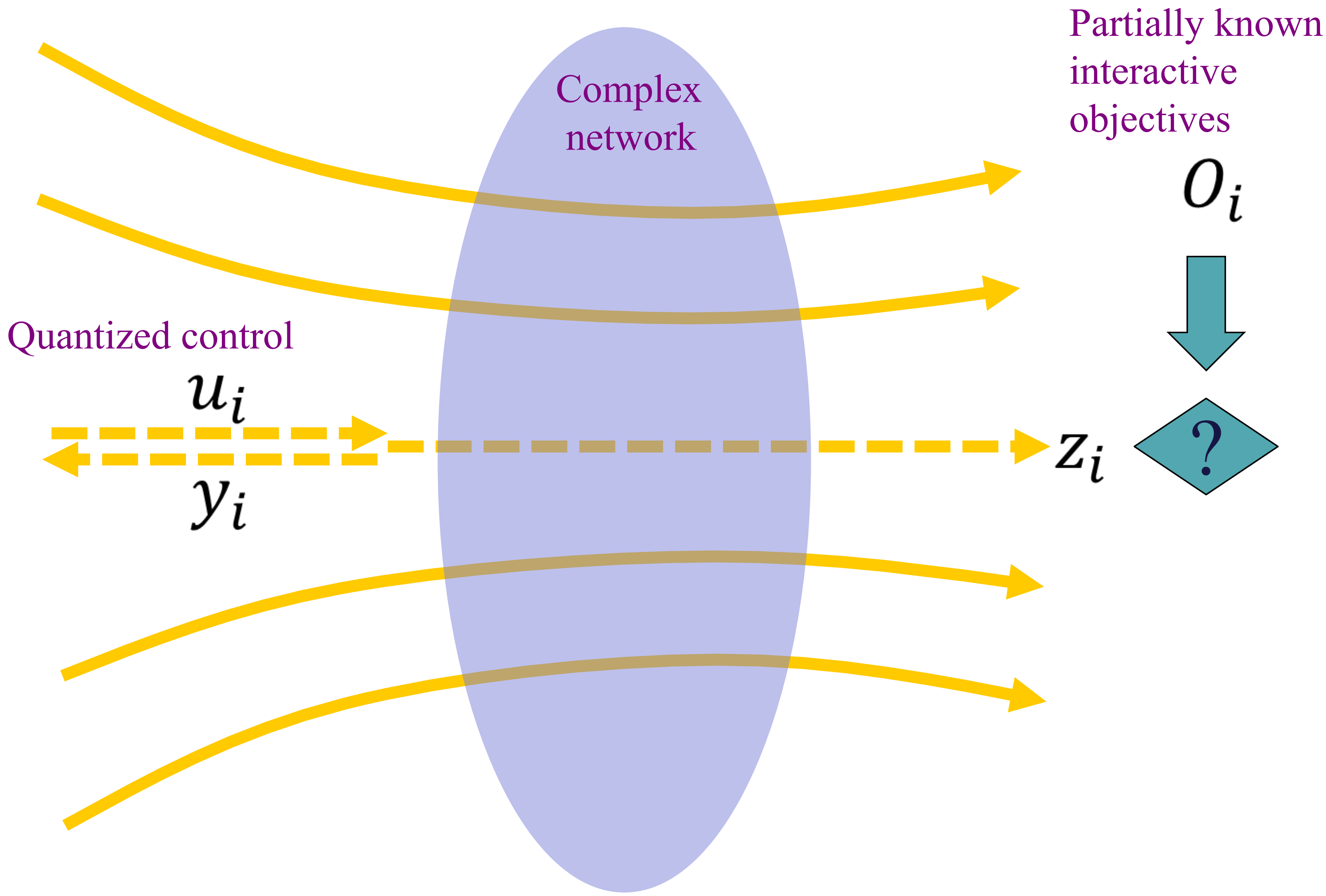
- Theorem: If a feasible solution exists, the tri-state algorithm converges to a solution, $\hat{\mathbf{P}}$, with the property:

$$\delta^{-1}\gamma_i \le \Gamma_i(\hat{\mathbf{P}}) \le \delta\gamma_i$$

- A simple version of information-limited control

Some salient observations

- Users can join and leave the network
- Observer and controller are not co-located
- Setting of information-based control
  - Communication bandwidth limited
- The objective of user i is represented by γ_i, known only to user i



Key features of an NOI

Network of Open Interaction

- State evolution

$$\frac{d\mathbf{x}(t)}{dt} = \mathbf{f}(\mathbf{x}(t), \mathbf{u}_1(t), \ldots, \mathbf{u}_L(t)), \quad \mathbf{x}(0) = \mathbf{x}_0 \quad \text{(S)}$$

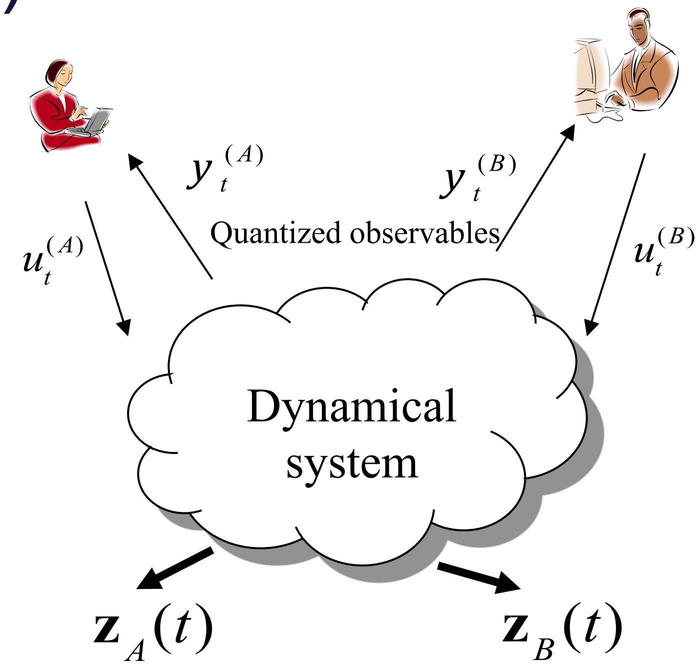
- Observation and (quantized) control

$$\mathbf{y}_i(t) = \mathbf{h}(\mathbf{x}(t)), \ \mathbf{u}_i(t) = \mathbf{q}_i(\mathbf{y}_i),$$
$$i = 1, \ldots, L$$

- Output

$$\mathbf{z}_i(t) = \mathbf{g}_i(\mathbf{x}(t)), \quad i = 1, \ldots, L$$

- ***What are the objectives?***



More about the objectives

- Single objective versus multiple objectives
- Competitive versus cooperative
- Time-constant or time-varying
- Honest agents versus non-honest agents

A Network of Open Interaction is a multiple objective, cooperative, time-varying, honest distributed system.

Issue with the objectives

- The objectives are only partially known and could be dynamically evolving.
- In addition to uncertainties due to state disturbances and observation noises, there are uncertainties due to multiple objectives.
- Assume agents select their choices with a known distribution and the choices remain unchanged.

Choice-based action system

2. Choice-based rendezvous

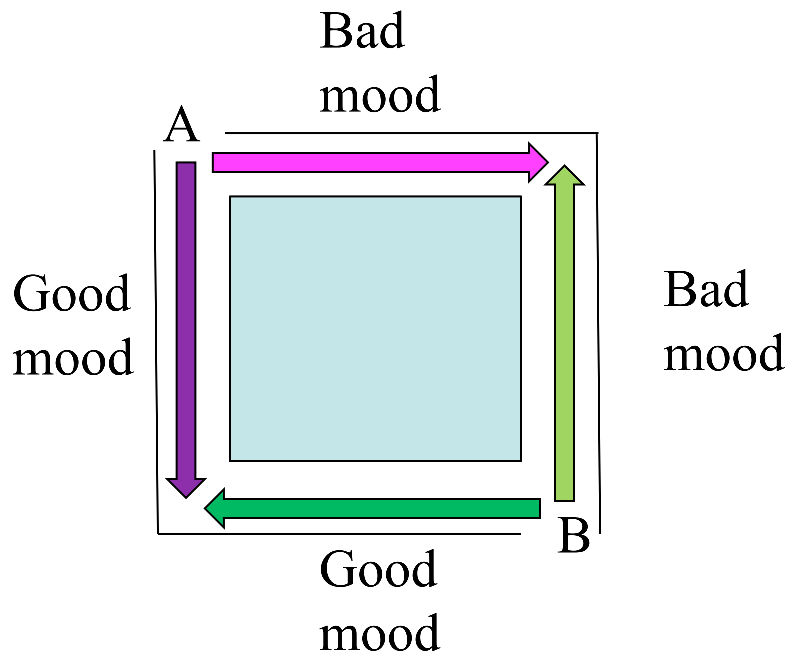
- Standard rendezvous problem: define path planning strategies for a mother and child separated in a park can meet again.
- With multiple choices: Alice and Bob jog in the same park at the same time every day, is there a path planning strategy to ensure the following?

Alice \ Bob	Good mood	Bad mood
Good mood	Meet	Won't meet
Bad mood	Won't meet	Meet

or $$\mathbf{H} = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

2. Choice-based rendezvous cont.

$$\mathbf{H} = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$



But how about the following target matrix:

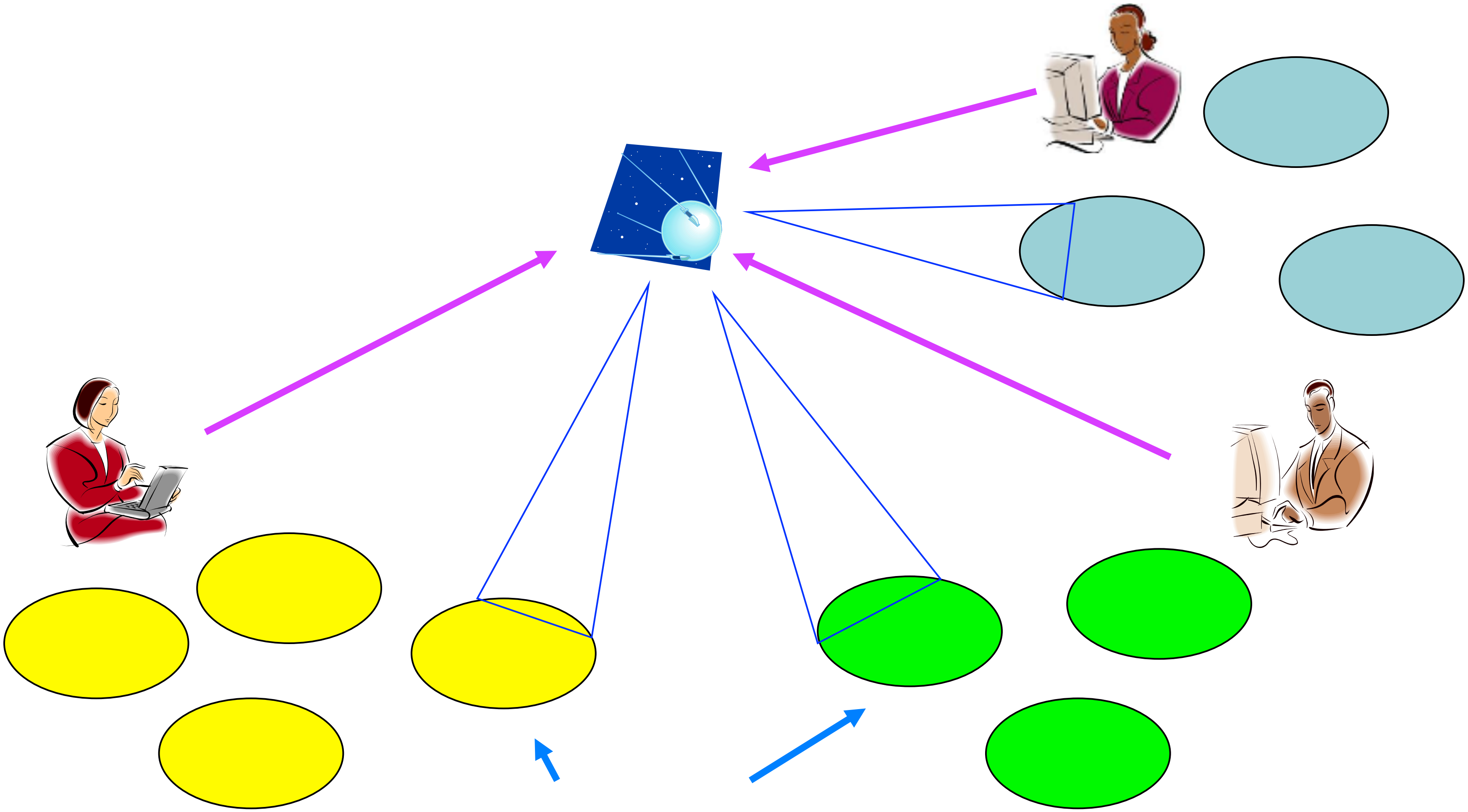
$$\mathbf{H} = \begin{bmatrix} 1 & -1 \\ -1 & -1 \end{bmatrix}$$

?

3. Interactive objectives - Salsa

Video courtesy of Baillieul and Oczimder

4. Sensor placing problem



Regions requiring attention

Model for sensor positioning

- Assuming a linear dynamic system

$$\frac{d\mathbf{x}(t)}{dt} = \mathbf{A}\mathbf{x}(t) + \sum_{i=1}^{L} \mathbf{b}_i \mathbf{u}_i(t), \qquad \mathbf{x}(0) = \mathbf{x}_0 \in \mathbf{R}^4 \qquad (\Lambda)$$

- Observation and (quantized control)

$$\mathbf{y}_i(t) = \mathbf{x}(t), \quad \mathbf{u}_i(t) = \mathbf{q}_i(y_i)$$

- Agent i has a number of locations to monitor

$$\{\mathbf{P}_{i,1}, \mathbf{P}_{i,2}, \ldots, \mathbf{P}_{i,N_i}\}$$

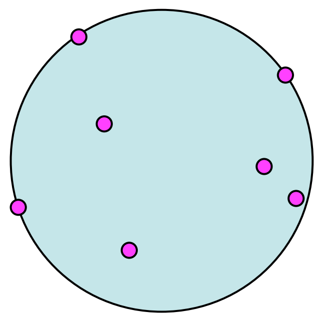
- Aim to position the sensor according to the agents' choices ***without a central coordinator***

Two definitions of target position

1. Sensor located at the arithmetic means of the points of interest

$$H_{i_1,\ldots,i_L} = \frac{1}{L}\left(P_{1,i_1} + \cdots + P_{L,i_L}\right)$$

2. Sensor located at the center of the minimum covering circle



- In both cases, the target positions can be represented by a $N_1 \times N_2 \cdots \times N_L$ tensor.
- In a two-agent case, by an $N_1 \times N_2$ matrix, **H.**

The basic question

- Can the sensor be controlled jointed by the agents **without** a central coordinator?
- If yes, does it require some form of signaling among the agents?
- How much ***more*** control cost is needed?

Two extreme approaches

- No communication versus full-communication
- *Is it possible to achieve multiple objectives with no communication between agents?*
- Choice-based target realization problem (CBTRP): Consider the system Λ. Given a target matrix, **H**, a set of choices and a termination time, τ, are there control sets, $\{u_{1,1},\ldots,u_{1,N_1}\},\ldots,\{u_{L,1},\ldots,u_{L,N_L}\}$ so that

$$H_{i_1,\ldots,i_L} = \mathbf{x}(\tau, u_{1,i_1},\ldots,u_{L,i_L})$$

Basic result 1

- **Theorem** [WB] [GLW]: Assuming the linear system Λ is jointly controllable, then a target matrix is realizable if and only if for any $i_l, i_l' \in \{1, \ldots, N_l\}$ and any $i_m, i_m' \in \{1, \ldots, N_m\}$

$$H_{i_1 i_2 \cdots i_l \cdots i_m \cdots i_L} - H_{i_1 i_2 \cdots i_l \cdots i_m' \cdots i_L} = H_{i_1 i_2 \cdots i_l' \cdots i_m \cdots i_L} - H_{i_1 i_2 \cdots i_l' \cdots i_m' \cdots i_L}$$

[■1&2@5&6]

- For two-agent cases, it means

[■1&2@5&5]X

$$H_{i,l} - H_{i,k} = H_{j,l} - H_{j,k}$$

- Matrices satisfying the condition are called *compatible*.

Basic result 2

- **Theorem** [GLW]: Consider the quadratic control cost

$$J = \sum_{l=1}^{L} \int_0^{\tau} \mathbf{u}_l^T \mathbf{u}_l \, dt$$

If the system Λ is individually controllable and **H** is compatible, the optimal control sets are defined by:

$$\hat{\mathbf{u}}_{m,j}(t) = \begin{cases} \mathbf{B}_1^T e^{-\mathbf{A}^T t} \mathbf{W}_1^{-1} \left(e^{-\mathbf{A}\tau} H_{1\cdots 1} - \mathbf{x}_0 - \sum_{k=2}^{L} \mathbf{W}_k \hat{\mathbf{P}}_k^1 \right), \\ \mathbf{B}_m^T e^{-\mathbf{A}^T t} \mathbf{W}_m^{-1} \left(e^{-\mathbf{A}\tau} (H_{1\cdots i_m \cdots 1} - H_{1\cdots 1}) + \hat{\mathbf{P}}_m^1 \right), \quad m = 2, \ldots, L \end{cases}$$

for some $\mathbf{W}_i$ and $\hat{\mathbf{P}}_m^1$ that can be explicitly defined.

Basic result 3

- Control cost is lower bounded by the averaged control cost of corresponding single target control problems.
- Consider a two-agent scalar example
- $\mathbf{A}=0$, $\mathbf{b}_i=1$, a general 2-by-2 compatible $\mathbf{H}$

$$J_{\min} = \frac{1}{4\tau}[(H_{11}-x_0)^2+(H_{22}-x_0)^2+\frac{1}{2}(H_{12}-H_{21})^2]$$

$$J_{avg} = \frac{1}{8\tau}[(H_{11}-x_0)^2+(H_{22}-x_0)^2+(H_{12}-x_0)^2+(H_{21}-x_0)^2]$$

$$J_{\min}-J_{avg} = \frac{1}{8\tau}[(H_{11}-x_0)^2+(H_{22}-x_0)^2-2(H_{12}-x_0)(H_{21}-x_0)]\geq 0$$

$2J_{avg} \geq J_{\min}$ (equality holds iff $H_{12}=H_{21}$ since $H_{11}+H_{22}=H_{12}+H_{21}$)

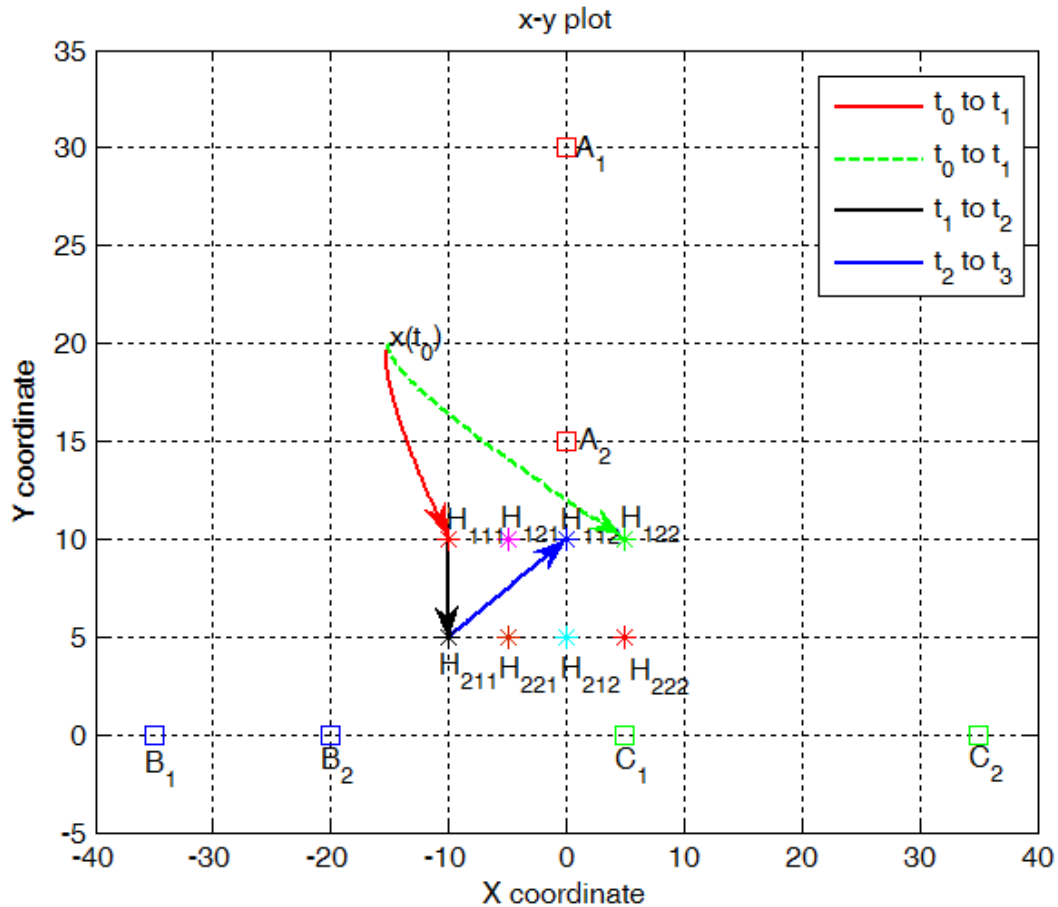
Sensor located at the arithmetic mean

- The target matrix, **H,** with entries defined by

$$H_{i_1,\ldots,i_L} = \frac{1}{L}\left(P_{1,i_1} + \cdots + P_{L,i_L}\right)$$

is compatible and optimal control laws can be derived.

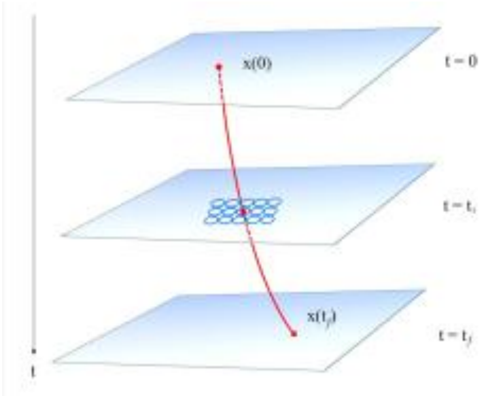
$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{b}_1 = \mathbf{b}_2 = \mathbf{b}_3 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \quad \frac{d\mathbf{x}(t)}{dt} = \mathbf{A}\mathbf{x}(t) + \sum_{i=1}^{L} \mathbf{b}_i \mathbf{u}_i(t)$$



Optimal trajectory of sensor for a three-agent case

Sensor located at the center of minimal covering circle

- The target matrix is not compatible.
- Signaling is necessary.



- Two-phase solution:
  - First phase – target signaling
  - Second phase – target control
- Control cost *J* is a sum of the two phase costs

$$J = J_{Phase\,1} + J_{Phase\,2}$$

- Cost of the first phase is dependent on the number of possible choices and the detection error margin.

A modified 2-stage system (LW)

- Consider $x(t+1) = x(t) + u(t) + v(t) + w(t), \ x(0) = 0, t = 0, 1$

  $w(t)$ is Gaussian with variance δ^2
- Alice observes: $y_A(t) = x(t) + \eta_A(t)$
- Bob observes: $y_B(t) = x(t) + \eta_B(t)$

  $\eta_B(t), \eta_B(t)$ are Gaussian with variance ε^2
- Target matrix:

$$\mathbf{H} = \begin{bmatrix} R & R \\ R & -R \end{bmatrix}$$

- No terminal constraint but define control cost:

$$J = \frac{1}{4}\left(\sum_{i,j} (x(2, u_i, v_j) - H_{ij})^2 + \sum_i u_i^2 + \sum_j v_j^2 \right)$$

Comparing optimal control costs

- Linear controller for a single target, (R or $-R$),

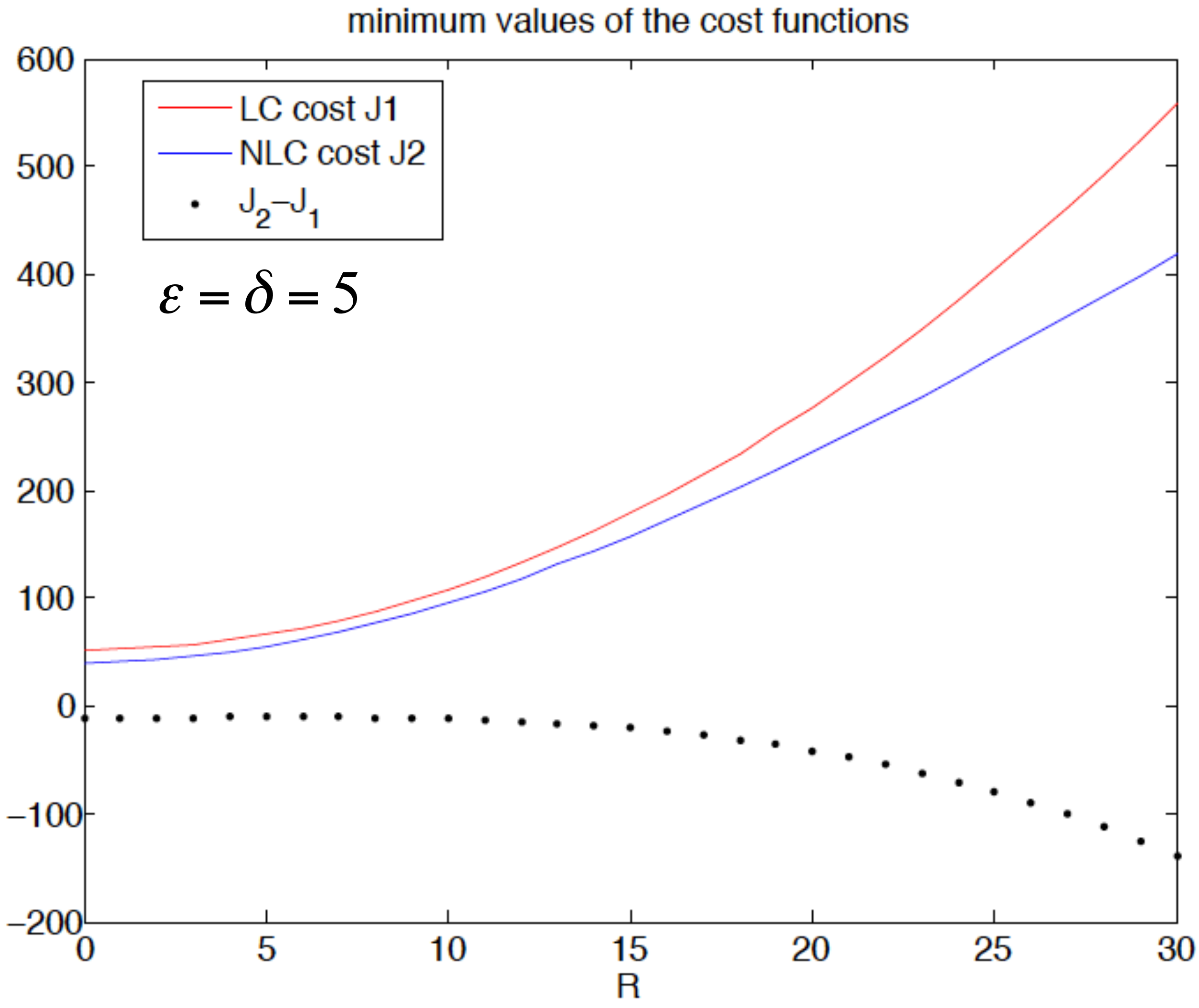
$$\text{Optimal cost} = \frac{R^2}{5} + \frac{124\varepsilon^2}{225} + \frac{4\delta^2}{3}$$

- Linear controller, multiple choices, optimal J

$$J^* = \frac{9R^2}{16} + \frac{19\varepsilon^2}{16} + \frac{3\delta^2}{2}$$

- Nonlinear controller using first stage to signal can achieve for large R and small noises

$$\tilde{J} \approx \frac{31R^2}{120} + \frac{4\varepsilon^2}{9} + \frac{4\delta^2}{3} < J^*$$



Optimal linear control (LC) versus nonlinear control NLC

Witsenhausen type counterexample

- This reminds us of the famous Witsenhausen counterexample
- Job coordination between a “weak” controller and a “blurry” controller implies linear feedback law suboptimal.
- Job coordination between “objective signaling” and “objective control” also implies linear feedback law suboptimal.

Optimal control

- Two-phase scheme is suboptimal in general
- Optimal strategies will combine signaling and control
- How much signaling?
- Possible to define *control communication complexity* by extending the Andrew Yao's communication complexity
- A protocol consists of a number of observation and action rounds. Classify protocol by the number rounds used. Zero-round is essentially open loop.

Results for nonlinear dynamical systems

A nonlinear system example

- Brockett-Heisenberg Integrator (BHI)

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = \begin{pmatrix} u_A \\ u_B \\ u_B x - u_A y \end{pmatrix}, \quad \begin{pmatrix} x(0) \\ y(0) \\ z(0) \end{pmatrix} = \begin{pmatrix} x_0 \\ y_0 \\ 0 \end{pmatrix}$$

$$h_A(t) = h_B(t) = F(t) = z(t)$$

- Interested only in output in the z-axis
- Consider only loop control functions on $[0,1]$ that produce close curves in the (x, y) plane
- Well known that $z(1) = \pm 2s$

s – the area enclosed by the curve $(x(t), y(t))$.

Bilinear I/O systems

- BHI is a special case of a system with bilinear input-output map, that is $F(u,v)$ is a bilinear functional in u and v.
- Given an orthonormal basis $\{\mathbf{e}_1, \mathbf{e}_2, \ldots\}$ F can be represented as an infinite matrix; u and v, as infinite dimensional vectors.
- The rank of F as a matrix is independent of the choice of the basis.

Brockett Heisenberg Integrator

- If the control functions are expressed in the basis

$$\{\sqrt{2}\sin(2\pi t), \sqrt{2}\cos(2\pi t), \sqrt{2}\sin(4\pi t), \sqrt{2}\cos(4\pi t), \ldots\}$$

- Then,

$$\mathbf{F}_B = \frac{1}{\pi}\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & \cdots \\ 0 & 1 & 0 & 0 & 0 & 0 & \cdots \\ 0 & 0 & 2^{-1} & 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & 2^{-1} & 0 & 0 & \cdots \\ 0 & 0 & 0 & 0 & 3^{-1} & 0 & \cdots \\ 0 & 0 & 0 & 0 & 0 & 3^{-1} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

- Question: find control sets $\{u_i\}_{i=1}^m, \{v_j\}_{j=1}^n$ to achieve a given *m*-by-*n* target matrix, **H**, while minimizing

$$I = \frac{1}{m}\sum_{i=1}^{m}\int_0^1 u_i^2(t)\,dt + \frac{1}{n}\sum_{i=1}^{n}\int_0^1 v_j^2(t)\,dt$$

Matrix formulation

- The problem can be represented as

$$\min\left(\frac{1}{m}tr\mathbf{U}\mathbf{U}^T+\frac{1}{n}tr\mathbf{V}\mathbf{V}^T\right)$$

subject to $\mathbf{H}=\mathbf{U}\mathbf{F}\ \mathbf{V}^T$

- System is feasible if and only if

$$rank\ \mathbf{F}\geq rank\ \mathbf{H}$$

So BHI can achieve any finite target matrix

Minimum control cost for BHI

- **Theorem** [WB]: Minimum control cost is:

$$\frac{2}{\sqrt{mn}} \sum_{k=1}^{\min(m,n)} r_k \sigma_k(\mathbf{H}), \quad r_k = \begin{cases} k/2 & \text{if } k \text{ is even,} \\ \lceil k/2 \rceil & \text{if } k \text{ is odd.} \end{cases}$$

- Consider an *orthogonal* n-by-n $\mathbf{H}$
- Control cost averaged over each entry:

$$\frac{2\pi}{n} \le \frac{2\pi}{n^2} \sum_{i,j=1}^{n} |H_{ij}| \le \frac{2\pi}{\sqrt{n}}$$

- For zero-round protocol, the minimal control cost:

$$\frac{\pi}{2}(n+2) \text{ for even } n, \quad \frac{\pi}{2}(n+2+\frac{1}{n}) \text{ for odd } n.$$

- Superlinear growth rate!

Information can greatly reduce control cost!

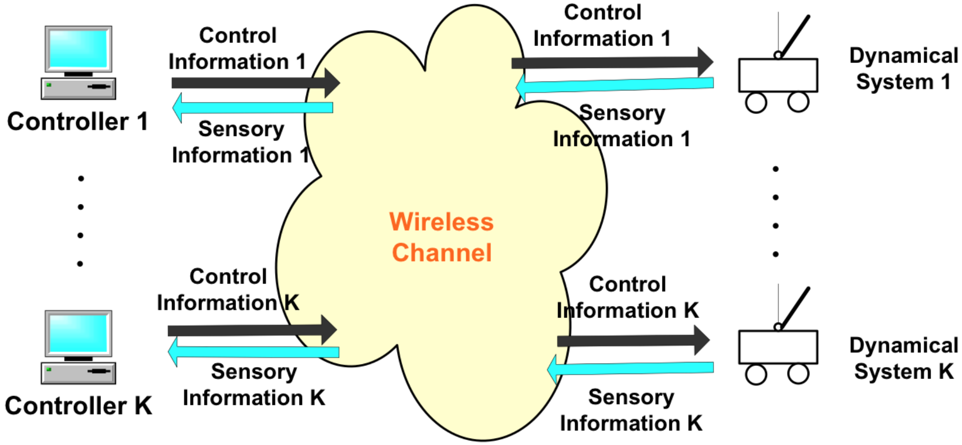
Control under resource contention

Resource sharing

- Since NOI allows open access, resource contention is unavoidable.
- Wireless communication channels are common contention hotspots.
- Traditional contention resolution approaches aim to optimize system throughput.
- For distributed control systems, delay is of primary concern.

Scheduling problem in NOI

- Scheduling in NCS has been extensively studied.
- For NOI, random access likely scenario.
- Lossy NOIs, controlling with missing observation or control instructions



TCP-like vs UDP-like

- Previous works model an individual linear system is by:

$$\mathbf{x}_{n+1} = \mathbf{A}\mathbf{x}_n + \alpha_n \mathbf{b}\mathbf{u}_n + \mathbf{w}_n,$$
$$\mathbf{y}_{n+1} = \mathbf{C}\mathbf{x}_{n+1} + \mathbf{v}_{n+1}$$

- α_n represents whether the control information has been properly received by the plant or not.
- If feedback control is based on prior history of the α_n's it is transmission control protocol (TCP)-like. Otherwise, it is datagram (UDP)-like.

Facts from the literature

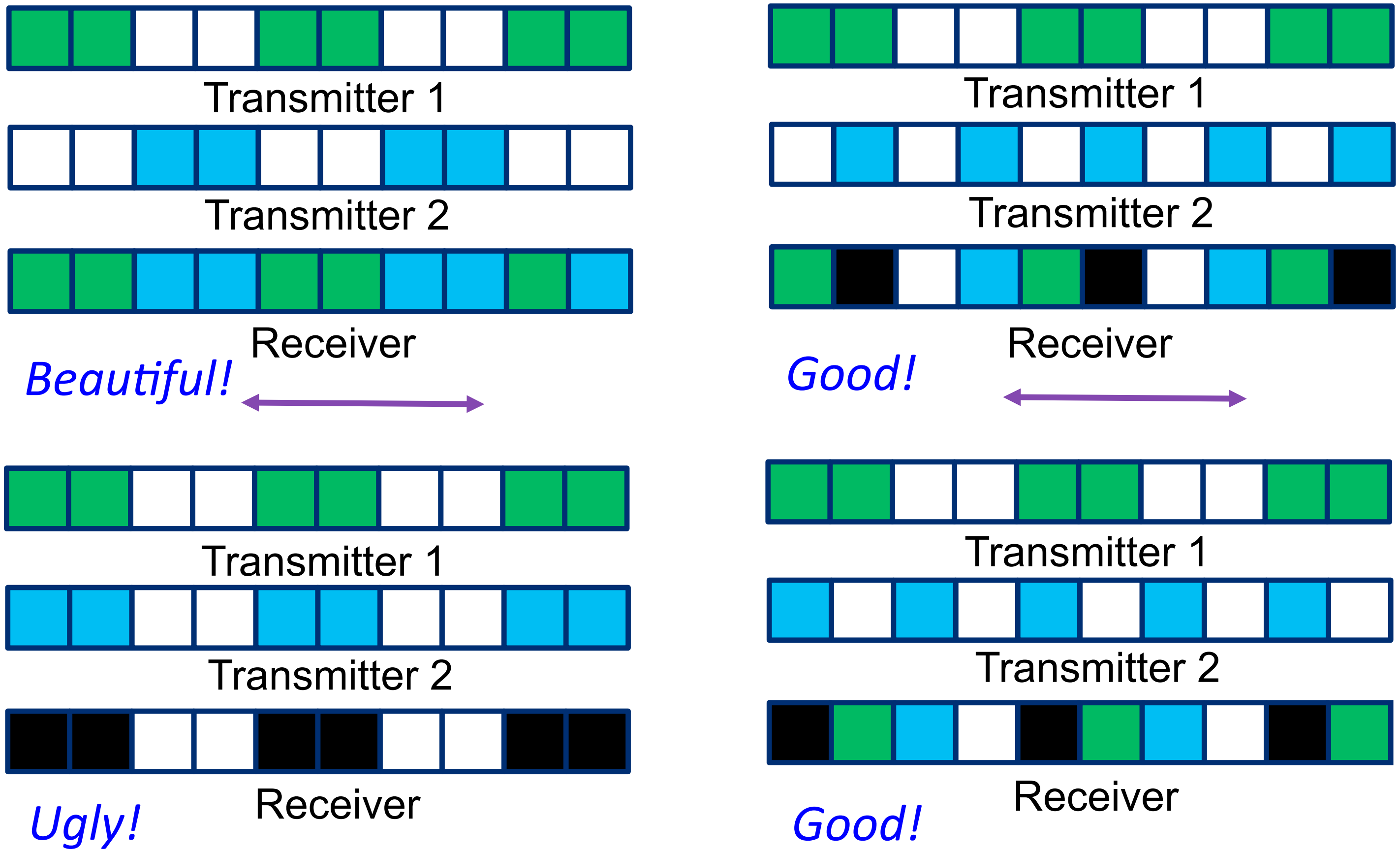
- Traditional media access control (MAC) protocols, such as, slotted ALOHA or CSMA etc leads to probabilistic α_n.
- Difficult to ensure TCP-like information structure (acknowledgment required)
- Separation of control and estimation works under TCP-like protocol but not for UDP-like protocol.
- UDP-like has smaller stable parameter region than TCP-like.

A different MAC paradigm: Protocol sequence based access control

Three controllers, one channel

Project team: H. Cheng, Y. S. Chen. W. S. Wong, Q. Yang, L. F. Shen, and J. Baillieul

Sequences of all shifts



Protocol sequences (PS) are periodic binary sequences with good Hamming cross-correlation properties under different shifts to provide a minimum goodput to each active user.

User-Irrepressible PS

- Assigns unique sequence to each user.
- If all active users transmitting according to the assigned sequences are guaranteed at least 1 collision-free packet in a period, the system is *user-irrepressible* (UI)
- E.g. one can construct UI sequences via the Chinese Remainder Theorem

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$\Leftrightarrow$$

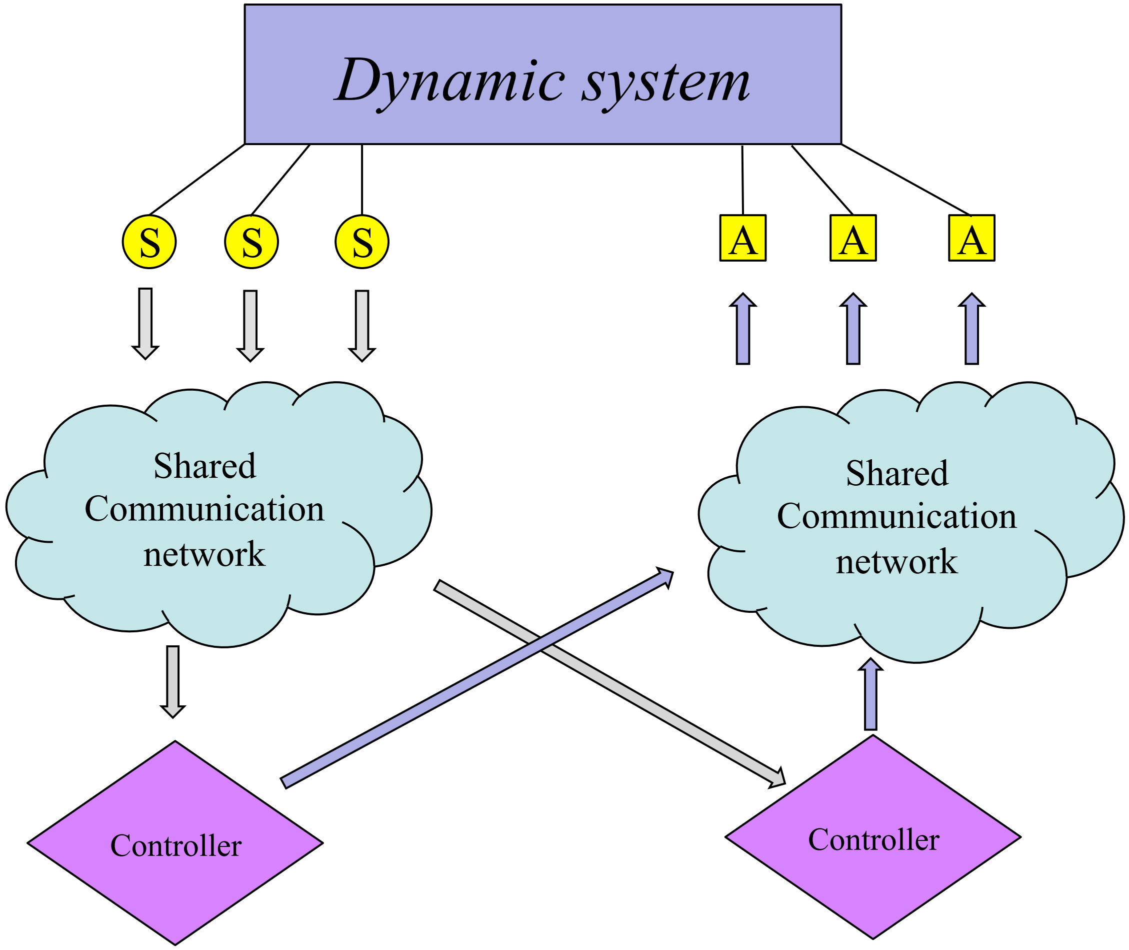
$$[1,0,0,0,0,0,1,0,0,1,0,0]$$

$$[1,1,1,0,0,0,0,0,0,0,0,0]$$

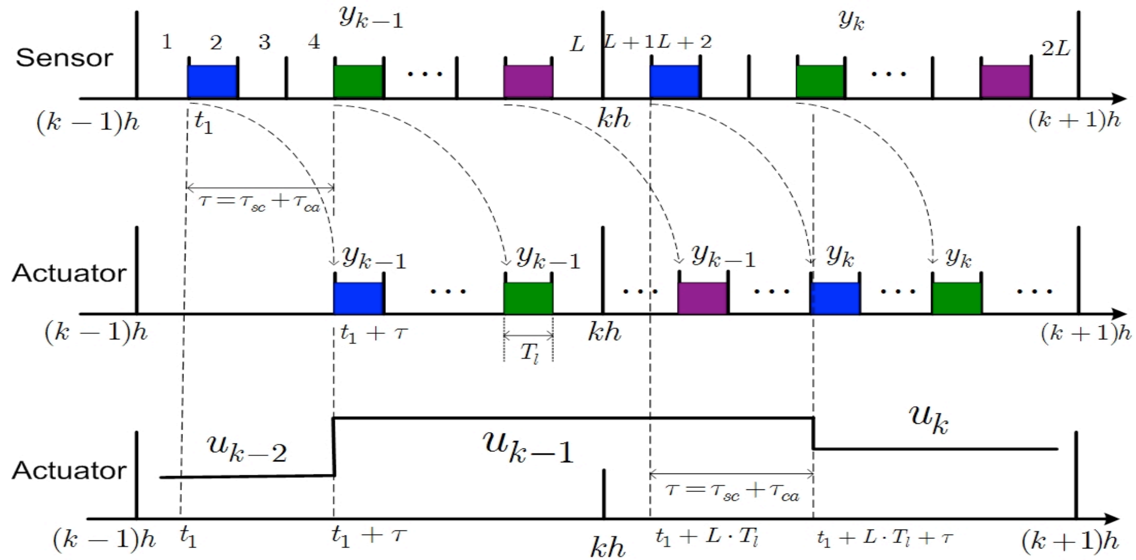
$$[1,0,0,0,0,1,0,0,0,0,1,0]$$

Protocol sequence based distributed control

S-sensor
A-actuator



Sensor event driven architecture



- Events driven by sensors and may conflict
- Use guard times to ensure contention-free control messages
- Delay assumed to be less than sampling time
- Multiple control messages on each sample data
- Hold-input control laws adopted

Protocol sequence (PS) control

- Under PS control, probabilistic loss is replaced by bounded deterministic delay

$$\mathbf{x}_{k+1} = \Phi\mathbf{x}_k + \Gamma_1(\tau^{sc},\tau^{ca})\mathbf{u}_{k-1} + \Gamma_0(\tau^{sc},\tau^{ca})\mathbf{u}_k + \mathbf{v}_k,$$

$$\mathbf{y}_k = \mathbf{C}\mathbf{x}_k + \mathbf{w}_k.$$

- Techniques for delayed systems apply
- Separation principle can be upheld.
- Optimal state estimator and simple suboptimal control can be derived using well-known results for quadratic cost function [CW]

$$J_N = E\{\sum_{k=0}^{N}(\mathbf{x}_k^T\mathbf{Q}_1\mathbf{x}_k + \mathbf{u}_k^T\mathbf{R}_1\mathbf{u}_k)\}$$

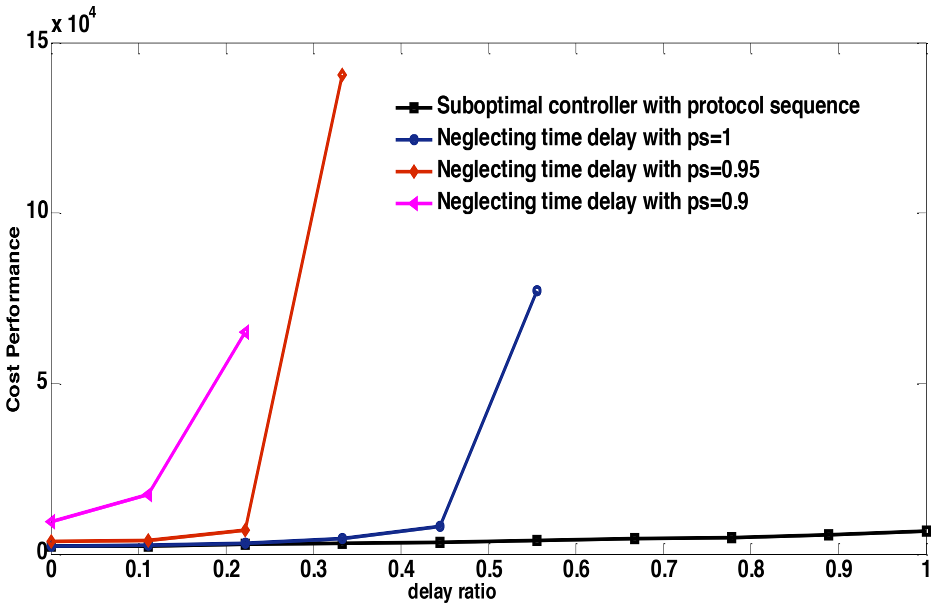
Numerical results

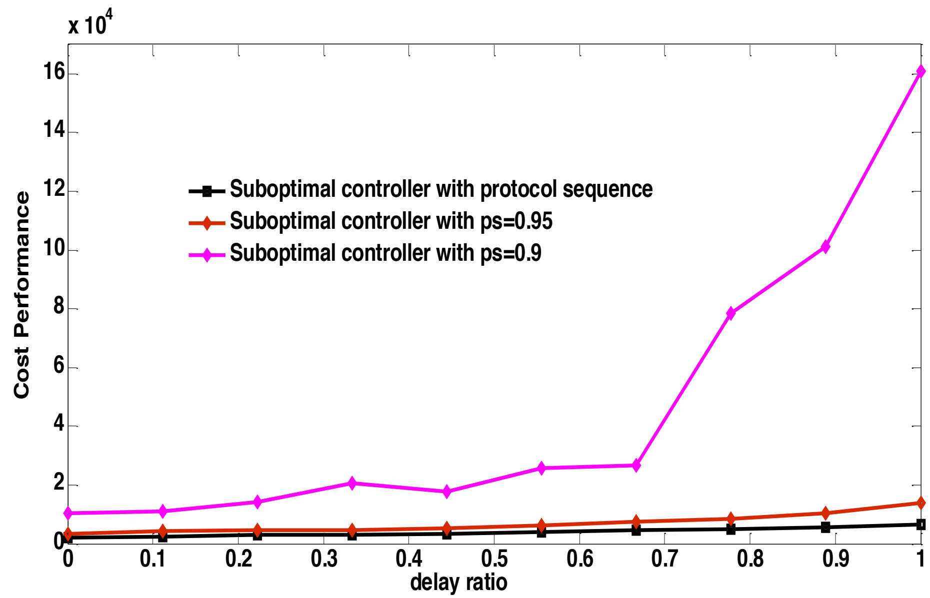
Basic system

$$\mathbf{x}[k+1]=\begin{bmatrix}1.1639,0.1054,0,0\\3.3650,1.1639,0,0\\-0.0021,-0.0001,1,0.1\\-0.0428,-0.0021,0,1\end{bmatrix}\mathbf{x}[k]+\begin{bmatrix}-0.0098\\-0.2018\\0.0031\\0.0614\end{bmatrix}u[k]+\mathbf{w}[k],$$

$$\mathbf{y}[k]=\mathbf{x}[k]+\mathbf{v}[k],$$

45 slots in one sampling interval





Cost as a function of time delays,

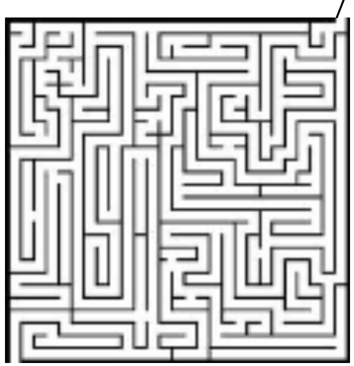
where delay $=\alpha * \text{sampling time}$, $0 \le \alpha \le 1$

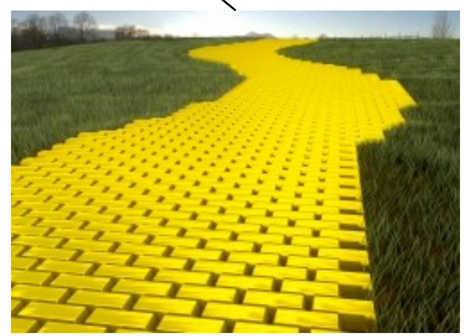
Concluding images

Learn to learn other players' objectives!

How to help Dorothy get home in the shortest time?

Home





Information is part of the optimization objective!

Google's Project Loon
Loon for All:

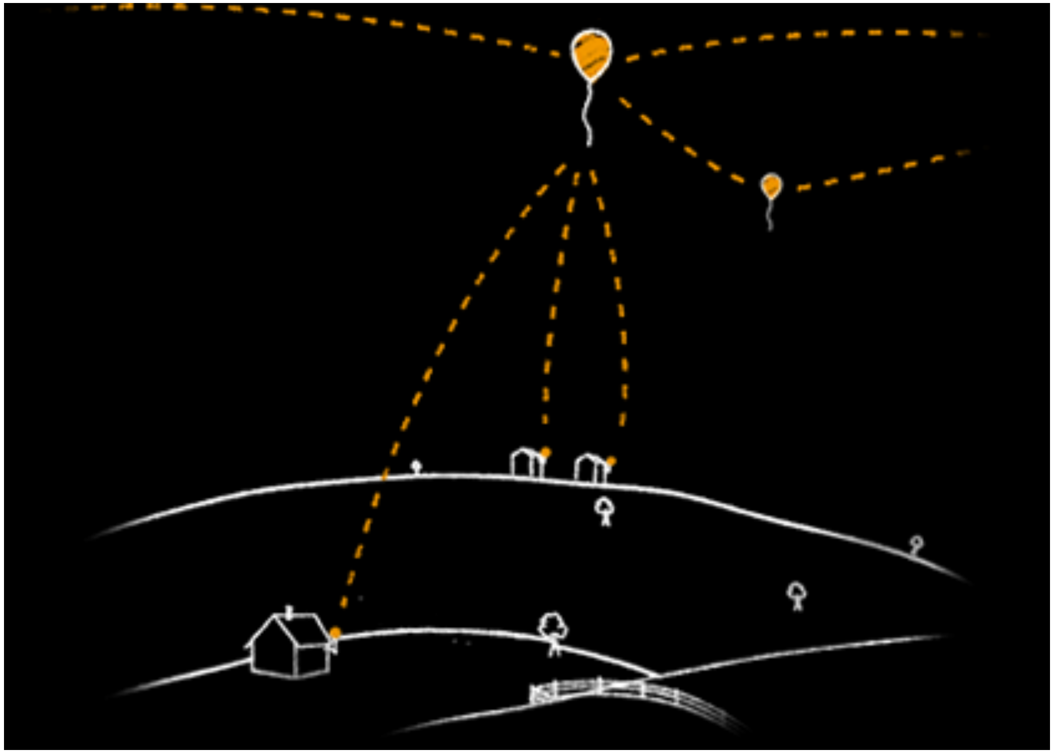


Image from http://www.google.com/loon/

Innovative applications in control and information!

For details:
http://www.ie.cuhk.edu.hk/wswong

THANK YOU